There and Back Again: A circuit extraction tale

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Quantum Week of Fun — September 23, 2020

Two paradigms of quantum computation

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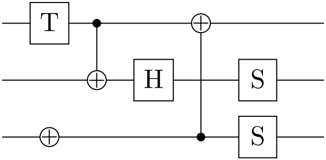
Gate based



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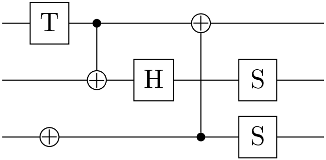

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- Successively apply small unitaries
- Measure qubits at the end

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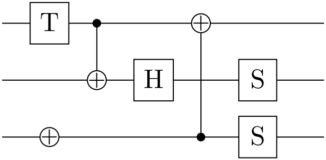

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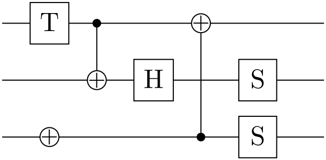

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Measurement based (MBQC)

- Prepare some highly entangled resource state
- Perform successive measurements
- Later measurement choices depend on earlier outcomes

Goal of this work

$$\begin{tikzcd}
\text{Patterns} \arrow[r, shift left, "\text{Translation}"] & \text{ZX} \arrow[l, shift left] \arrow[loop above, "\text{Optimisation}"] \arrow[r, hook, shift left, "\text{Circuit extraction}"] & \text{Circuits} \arrow[l, shift left]
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- ZX-calculus as a tool for translation.

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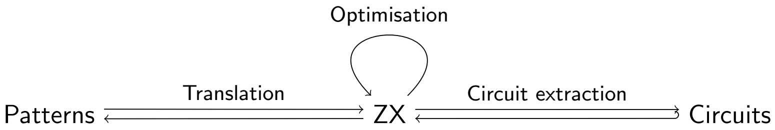

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- Optimise the number of qubits in measurement pattern.

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What gates are to circuits, *spiders* are to ZX-diagrams.

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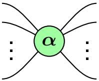
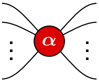
Z-spider	X-spider
$\lvert 0 \cdots 0 \rangle\langle 0 \cdots 0 \rvert$	$\lvert + \cdots + \rangle\langle + \cdots + \rvert$
$+ e^{i\alpha} \lvert 1 \cdots 1 \rangle\langle 1 \cdots 1 \rvert$	$+ e^{i\alpha} \lvert - \cdots - \rangle\langle - \cdots - \rvert$





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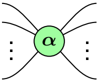
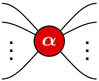
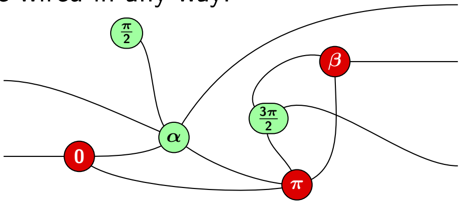
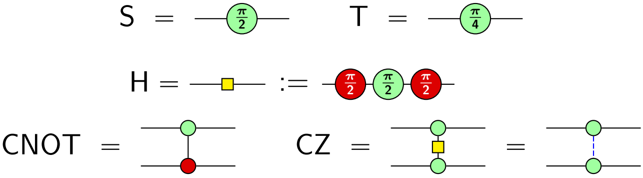
	

Spiders can be wired in any way:



Quantum gates as ZX-diagrams

Every quantum gate can be written as a ZX-diagram:



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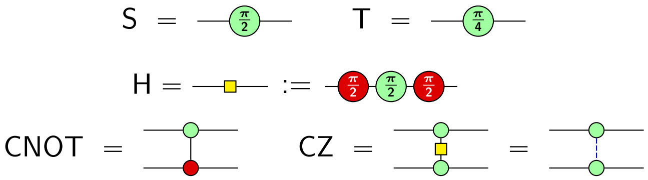
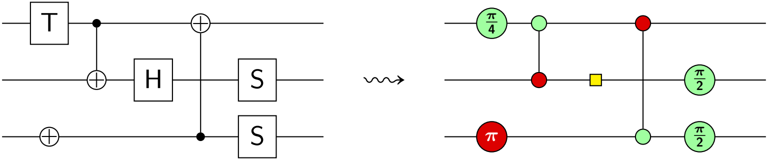
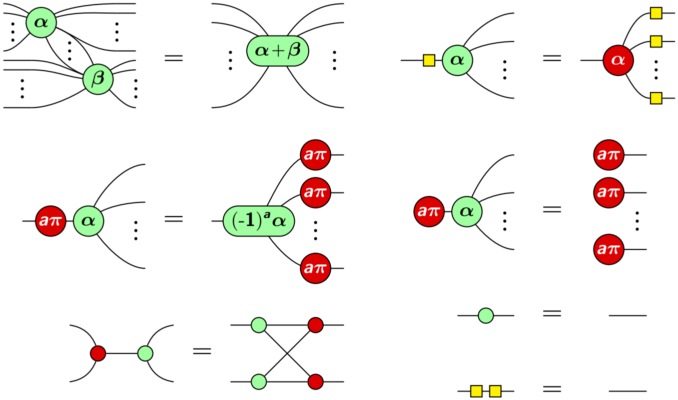




Rules for ZX-diagrams: The ZX-calculus



$\alpha, \beta \in [0, 2\pi]$, $a \in \{0, 1\}$

Universality and completeness

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Any linear map between qubits can be represented as a ZX-diagram.

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Any linear map between qubits can be represented as a ZX-diagram.

Theorem (Vilmart 2018)

If two ZX-diagrams represent the same linear map, then they can be transformed into one another using the previous rules (and one additional one).

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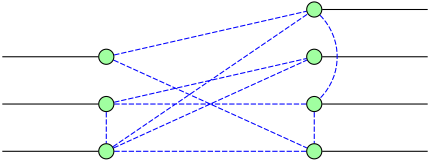
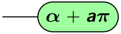
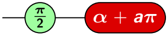
We will translate each of these components into the ZX-calculus.

(Open) graph states in the ZX-calculus



Measurements in the ZX-calculus

- XY-plane measurement: 

- XZ-plane measurement: 

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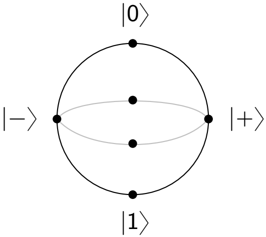
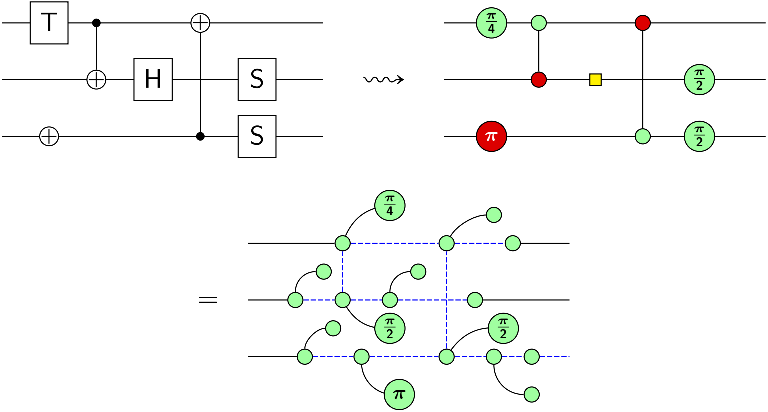


Circuit to measurement pattern



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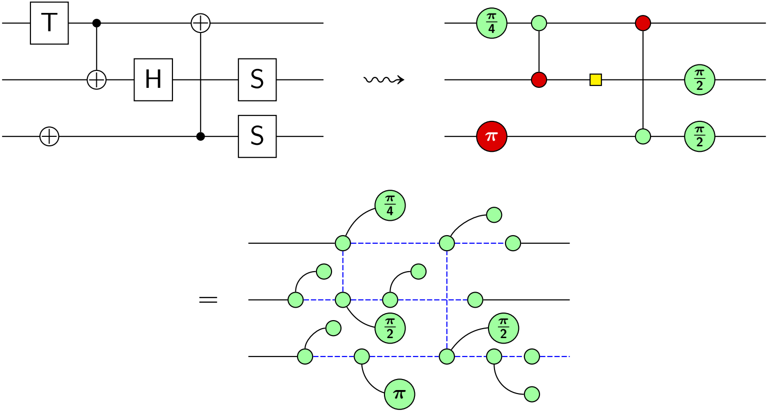

... but is this pattern deterministic?

Determinism and gflow

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The previous translation of circuits to patterns always has causal flow.

Main result

Theorem

There is an efficient algorithm to transform a measurement pattern with gflow into a unitary quantum circuit (i.e. no ancillae required).

This extends previous work by Duncan, Perdrix, Kissinger and vdW that only deals with XY-plane measurements.

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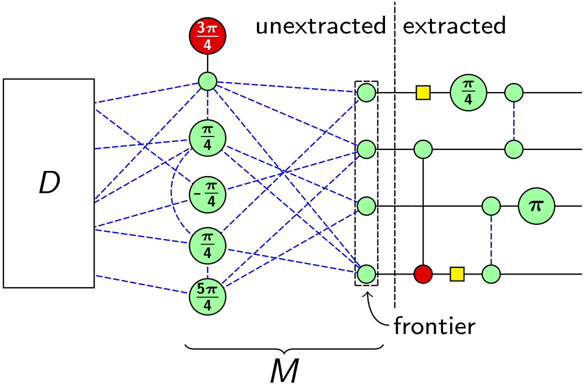

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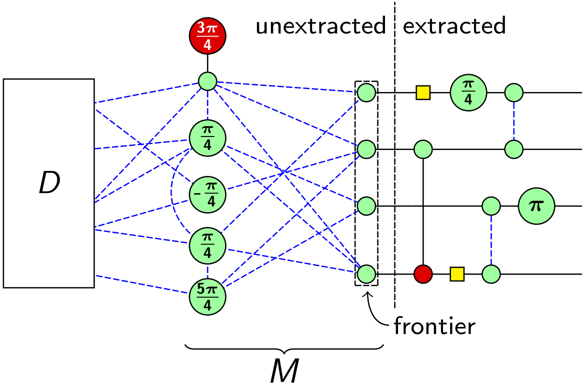

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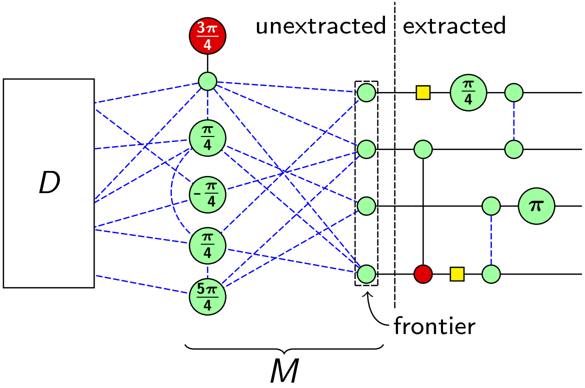
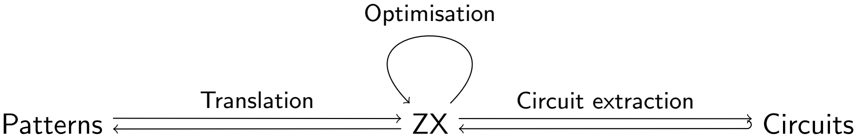


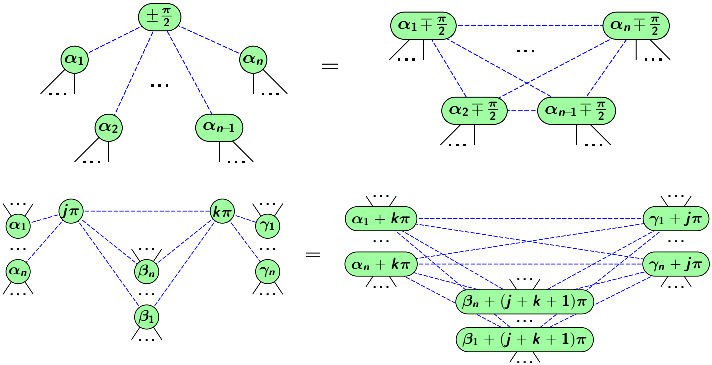
- Apply local rewrites to the unextracted part
- When the frontier vertices look like a circuit, move them to the extracted part

The overview again



Clifford vertex removal

Can remove any internal *Clifford* qubit from measurement pattern using *pivoting* and *local complementation*:



Theorem: These operations preserve existence of gflow.

Circuits to measurement patterns

Theorem

A n-qubit Clifford+T circuit with t T-gates can be efficiently transformed into a deterministic measurement pattern containing $t + O(n)$ qubits.

Conclusion

- We went there (circuits $\rightarrow$ measurement patterns)
- And back again (measurement patterns $\rightarrow$ circuits)
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Thank you for your attention

Backens, Miller-Bakewell, de Felice, Lobski & vdW 2020, arXiv:2003.01664. *There and back again: A circuit extraction tale*