Constructing quantum circuits with global gates

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Announcement: Quantum Pubquiz

- What: A quantum Pubquiz!
- When: Thursday 20:30CEST
- Where: Gathertown pub

No need to register, just show up :)

The problem

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- Ion trap quantum computers use *Mølmer-Sørensen* interaction that targets *many* qubits.
- Can we use this to our advantage somehow?
- As it turns out: yes!

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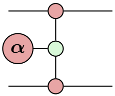
in ZX-calculus: $XX_{ij}(\alpha) \quad =$ 

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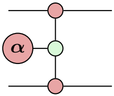

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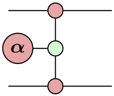

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- If S can be arbitrary, we say the interaction is *targeted*
- If S is necessarily *all* the qubits, it is *untargeted*.

Some observations

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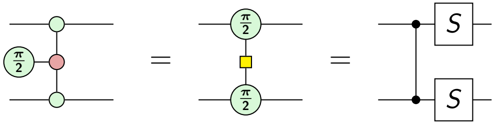

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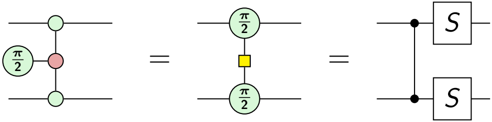

- Observation 4: A *global* CZ gate (GCZ) can be implemented using a single GMS gate + local Cliffords.

Compiling for targeted GMS gates

Input: circuit of Clifford gates and non-Clifford Z-phase gates.
Output: circuit of single-qubit gates and targeted GMS gates.

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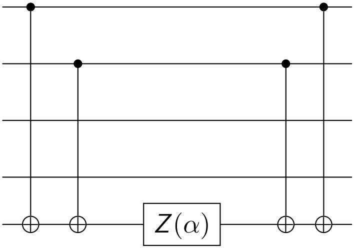
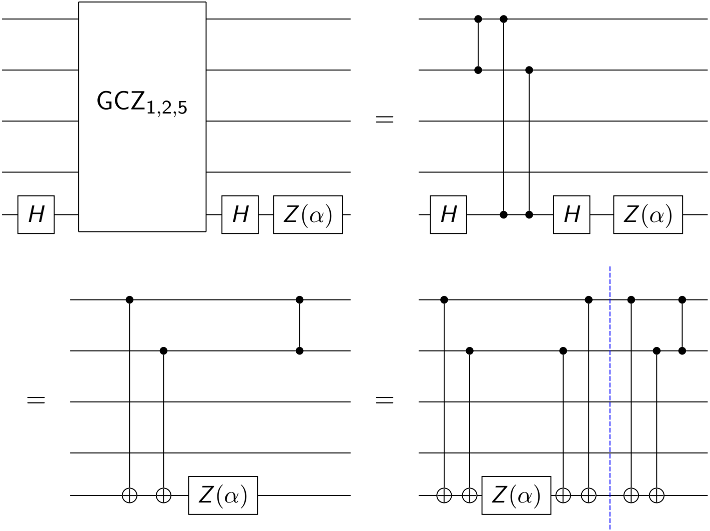
3. Implement $\vec{P}'(\alpha)$ using GCZ gate up to 'Clifford garbage'...

Phase gadget CNOT ladder

$$ZZIIZ(\alpha) \quad =$$



Phase gadget using GCZ gate



Compiling for targeted GMS gates contd.

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5. Synthesise remaining Clifford circuit.

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- Reduce the Clifford circuit to a normal form, i.e. Gottesman-Aaronson: C-P-C-P-H-P-C-P-C.

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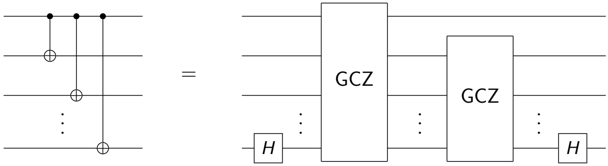
- Each CNOT 'fan-out' gate requires 2 GMS gates.



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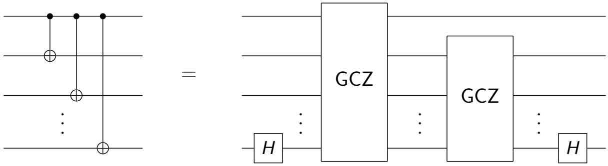

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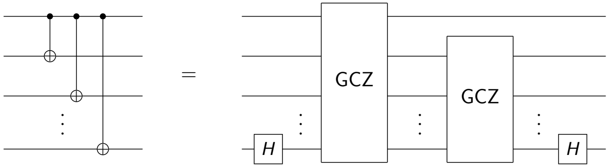

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- The above normal form then requires $12n - 18$ GMS gates.
- However: using 'GSLC' normal form H-S-CZ-CNOT-H-CZ-S-H we get $6n - 8$.

Compiling for targeted GMS gates

Theorem

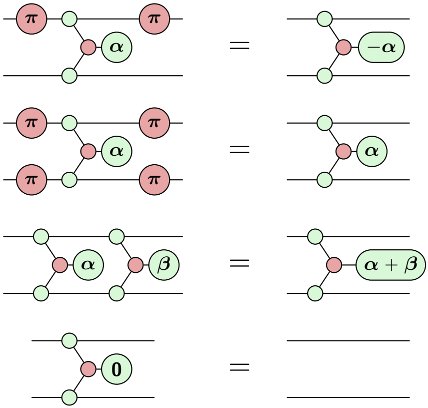
An n-qubit circuit consisting of

- Clifford gates
- and N non-Clifford Z-phase gates

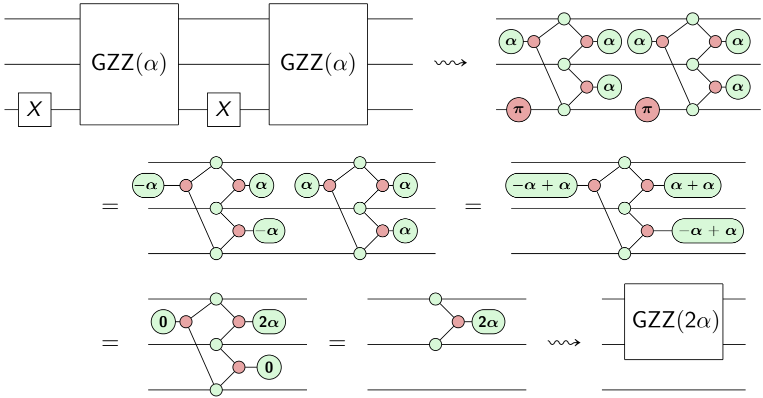
can be implemented using single qubit unitaries and at most $N + 6n - 8$ GMS gates.

Now to use untargeted GMS gates instead

Phase gadget identities

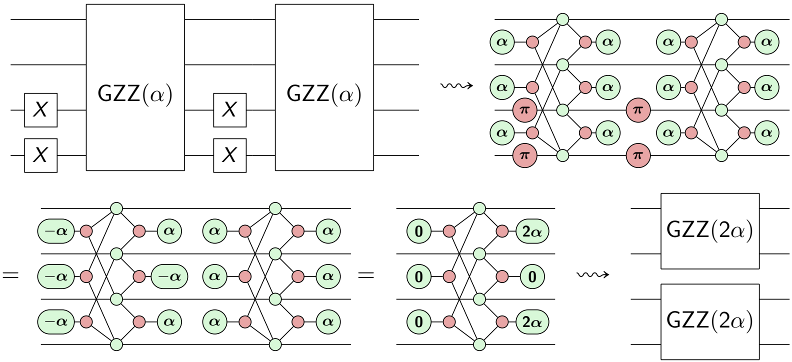


From global to local

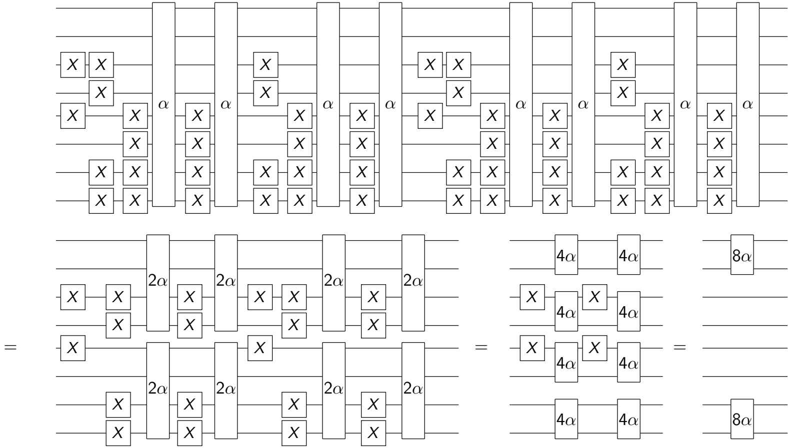


Identity from (Maslov & Nam, 2018)
To go from global on n qubits, to 'local' on k qubits requires 2^{n-k} GMS gates.

From global to more local



From 8-global to 2-local



Using the other method for the same result would require 128 GMS gates.

Cliffords using untargeted GMS gates

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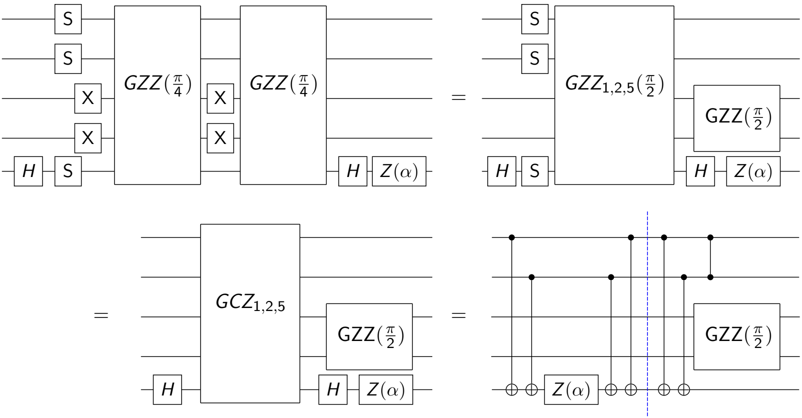
Note: this matches $O(n^2/\log(n))$ CNOT count for implementing an n-qubit Clifford.

Phase gadgets using untargeted GMS gates



Circuits to untargeted GMS gates

Theorem

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Future questions:

- Are these bounds tight?
- Is there a benefit to allow α to vary?

Thank you for your attention

vdW 2020, arXiv:2012.09061
Constructing quantum circuits with global gates

Maslov & Nam 2017, arXiv:1707.06356
Use of global interactions in efficient quantum circuit constructions

vdW 2020, arXiv:2012.13966
ZX-calculus for the working quantum computer scientist

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