PyZX: Quantum circuit optimization using the ZX-calculus

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Introduction

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- So:
  Optimizing fault tolerant QC means optimizing T-count.

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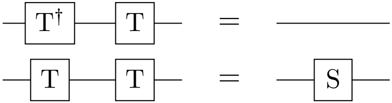
Existing heuristics fall basically in two categories.

Method 1: Commutation and Cancellation

Adjacent T gates become Clifford:



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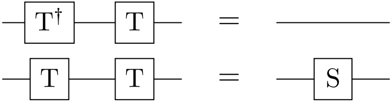

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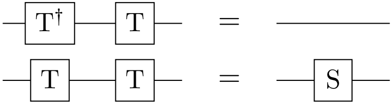

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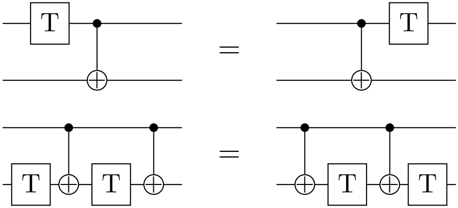

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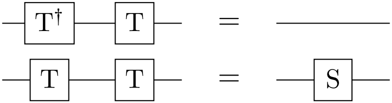

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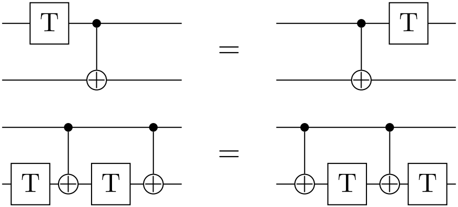

And loads more...

Method 2: Phase Polynomials

Circuits built out of CNOT and T gates can be written as

$$U\,|\mathbf{x}\rangle = e^{i\frac{\pi}{4}g(\mathbf{x})}\,|f_1(\mathbf{x}),\ldots,f_n(\mathbf{x})\rangle$$

where $\mathbf{x} \in \mathbb{Z}_2^n$ is a binary vector,
$g : \mathbb{Z}_2^n \to \mathbb{Z}_8$ is a polynomial and
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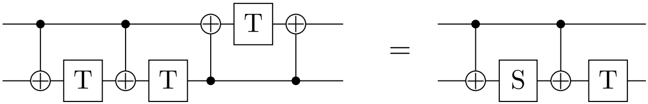

(but optimal T-count finding still seems to be in NP)

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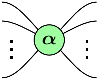
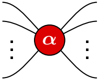
Enter the ZX-calculus

ZX-diagrams

- ZX-diagrams consist of two types of maps
- Z-spiders 

$:= \; |0\cdots0\rangle\langle0\cdots0| + e^{i\alpha}\,|1\cdots1\rangle\langle1\cdots1|$
- X-spiders 

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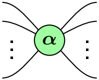
- Z-spiders 

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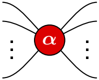
- X-spiders 

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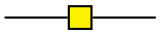
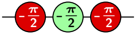
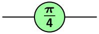
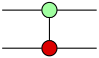
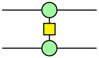
- By wiring these together, we can make arbitrary linear maps between qubits. For instance:

H =  = 

T = 

CNOT = 

CZ = 

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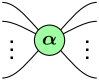
- Z-spiders 

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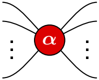
- X-spiders 

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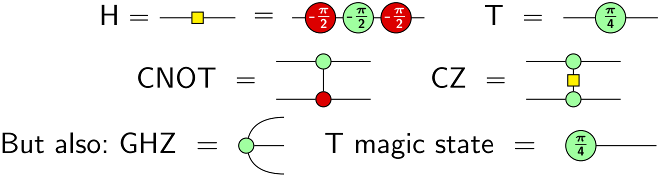
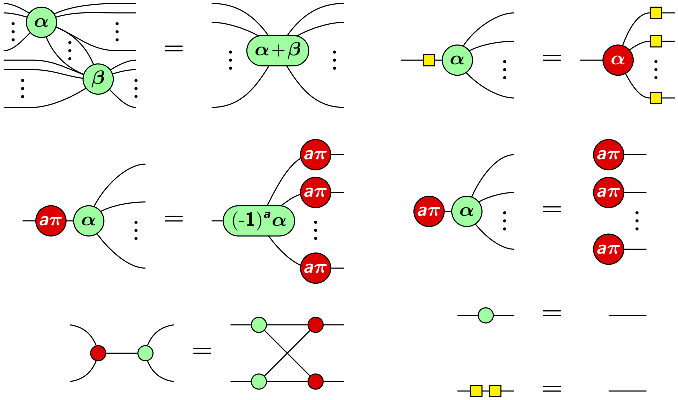


ZX-calculus



$\alpha, \beta \in [0, 2\pi], \; a \in \{0, 1\}$

Circuit optimization with the ZX-calculus

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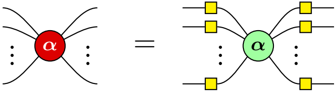
- Write your circuit as a ZX-diagram.
- Apply rewrite rules to simplify it.
- Turn the resulting diagram back into a circuit.

First steps: graph-like ZX-diagrams

- First turn all X-spiders into Z-spiders:



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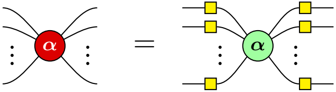

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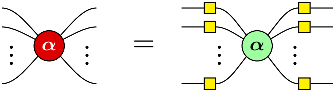

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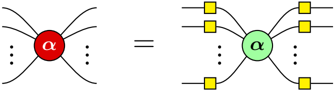
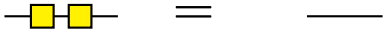


- Cancel all double Hadamards: 
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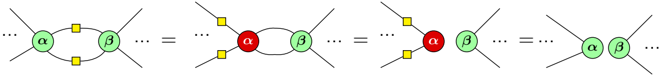

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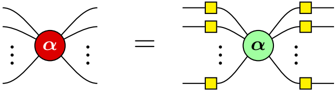

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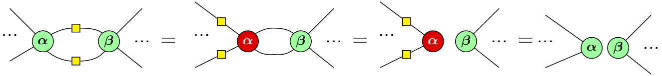
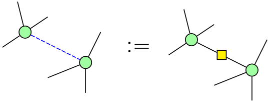
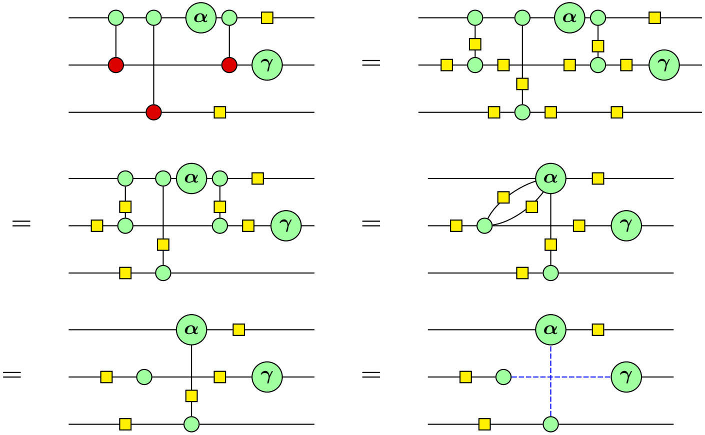


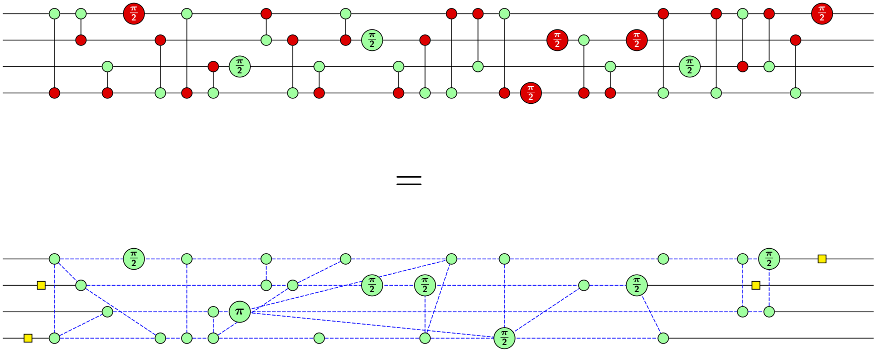
- Use new notation:



Example



More involved example



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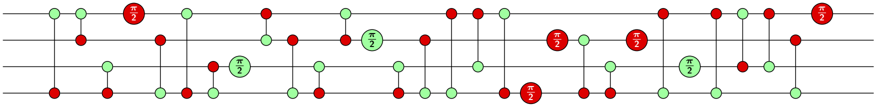
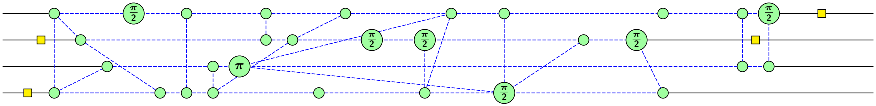


$$=$$



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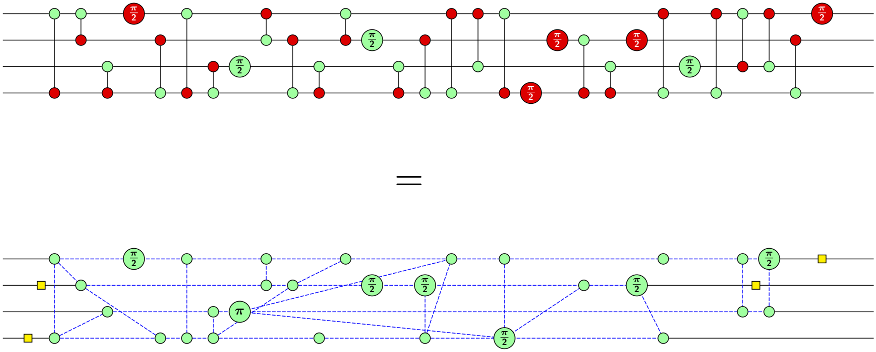

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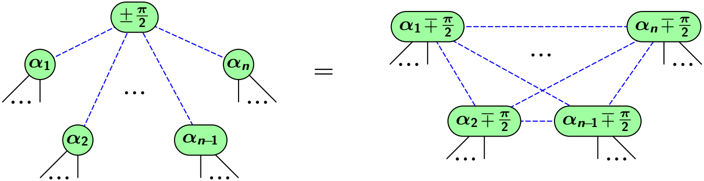
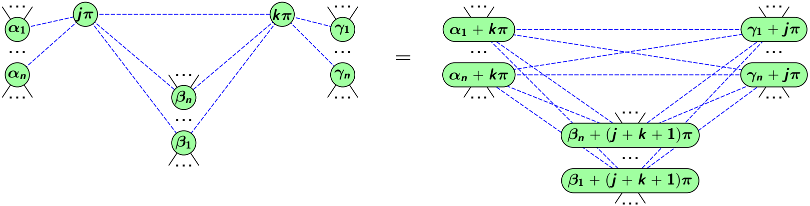
To simplify these diagrams, we want to remove as many interior vertices as possible.

Local complementation and pivoting





Simplification so far

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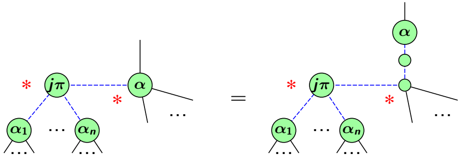

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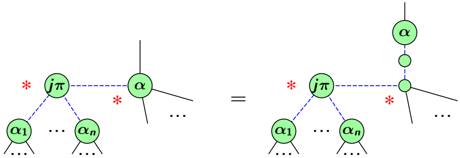
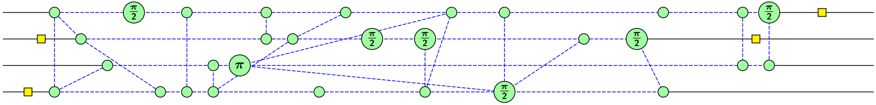


- If the original diagram was Clifford, then the simplified diagram has no internal spiders

Clifford example

Recall the example Clifford diagram:



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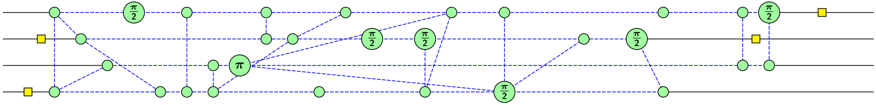
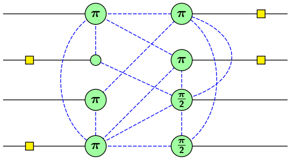
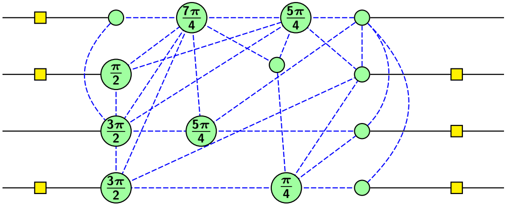


This can be reduced by the described procedure to:



Clifford+T example



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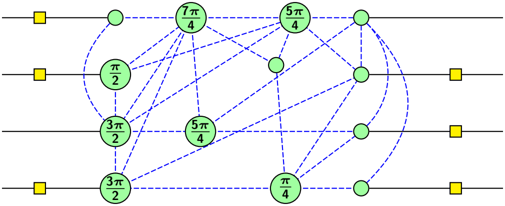

Question: How do we turn this into a circuit?

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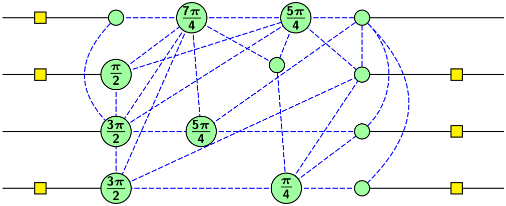

Question: How do we turn this into a circuit?
Answer: We use the fact that it has a *gFlow*.

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Informally, a gFlow associates an ‘arrow of time’ with a graph.
Circuits have a gFlow.

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Proposition

Local complementation and pivoting preserve gFlow

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Informally, a gFlow associates an 'arrow of time' with a graph. Circuits have a gFlow.

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Local complementation and pivoting preserve gFlow

Theorem

There is an efficient procedure that transforms a ZX-diagram with a gFlow into a circuit.

The simplification procedure

- We indeed have a circuit-to-circuit simplification procedure using the ZX-calculus.
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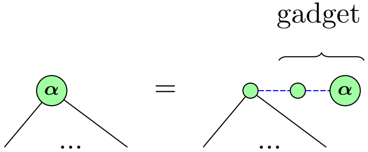
- It reduces Clifford circuits to a quasi normal-form.
- But: T gates never get removed by lcomp and pivoting.
- So:
  To do significant T-count optimization, we need to do better.

Gadgetization

We turn all non-Clifford spiders into *phase gadgets*:



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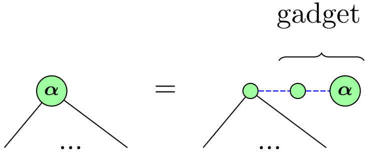
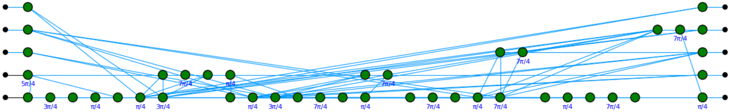


This makes the base of the gadget available for pivoting.

Example: Gadgetization and pivoting

After the first round of simplifications:



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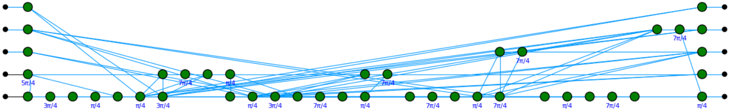

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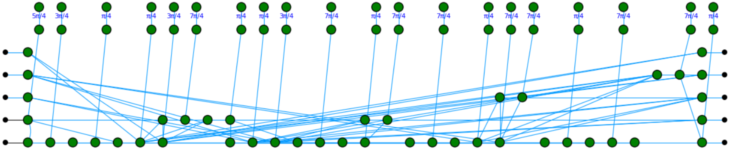

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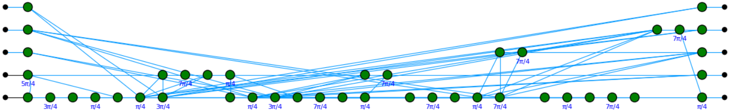
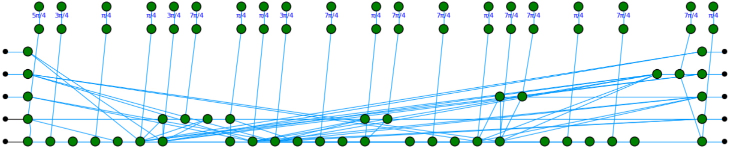
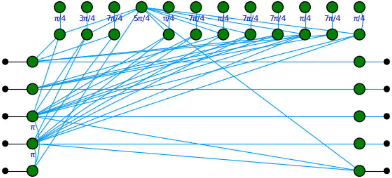


After gadgetization:

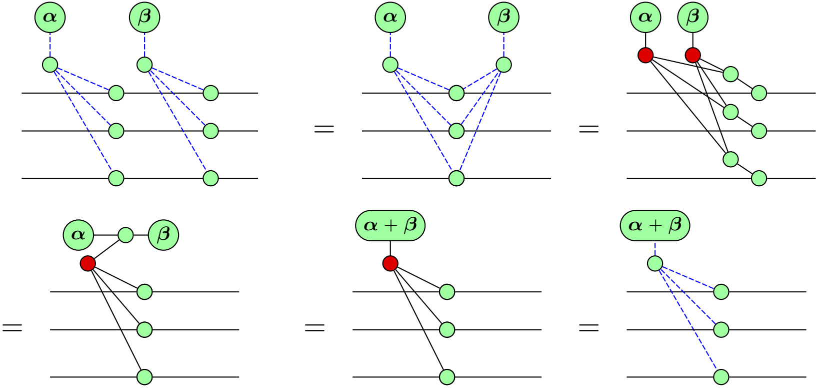


After pivots/lcomps:



Final step: phase gadget fusion

Whenever phase gadgets have the same set of neighbours, they can fuse:



Spider fusion + Local complementation + Pivoting + Gadgetization + Gadget fusion

=

State-of-the-art T-count optimization

Circuit extraction

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The good news: We have ‘heuristics’ that always seem to work.
The bad news: We don’t know why.

Demonstration of PyZX

Conclusion and future work

The Takeaway:

The ZX-calculus is very useful for
T-count optimization

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Open problems:

- Why does our circuit extraction work?

Conclusion and future work

The Takeaway:

The ZX-calculus is very useful for
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Open problems:

- Why does our circuit extraction work?
- How to use phase-polynomial methods on ZX-diagrams?

Conclusion and future work

The Takeaway:

The ZX-calculus is very useful for
T-count optimization

Open problems:

- Why does our circuit extraction work?
- How to use phase-polynomial methods on ZX-diagrams?
- Is the ZH-calculus useful?

Thank you for your attention