T-count optimization of quantum circuits using graph-theoretical rewriting of ZX-diagrams

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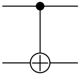
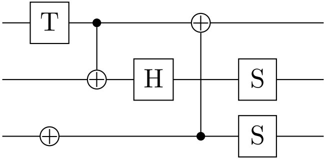
- So quantum circuits should contain as few gates as possible.
- Several important metrics:
  - Gate-depth
  - 2-qubit gate count
  - Number of T gates: *T-count* ($\mathrm{T} = R_Z(\frac{\pi}{4})$)

Circuit diagrams

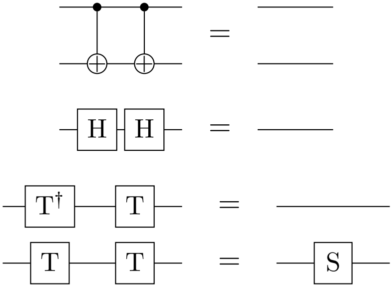
NOT $=$ 

CNOT $=$ 

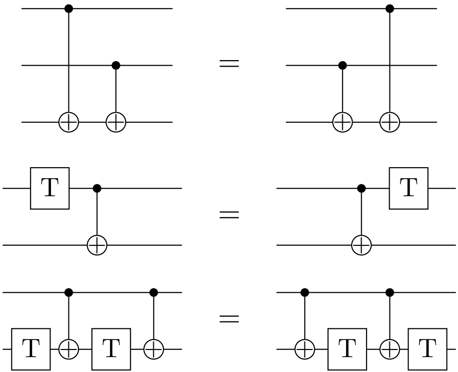
An example quantum circuit:



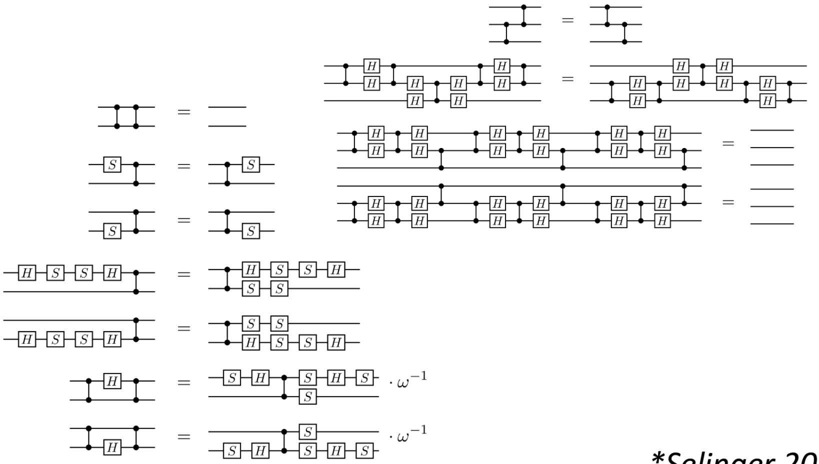
Circuit identities



Gate commutation

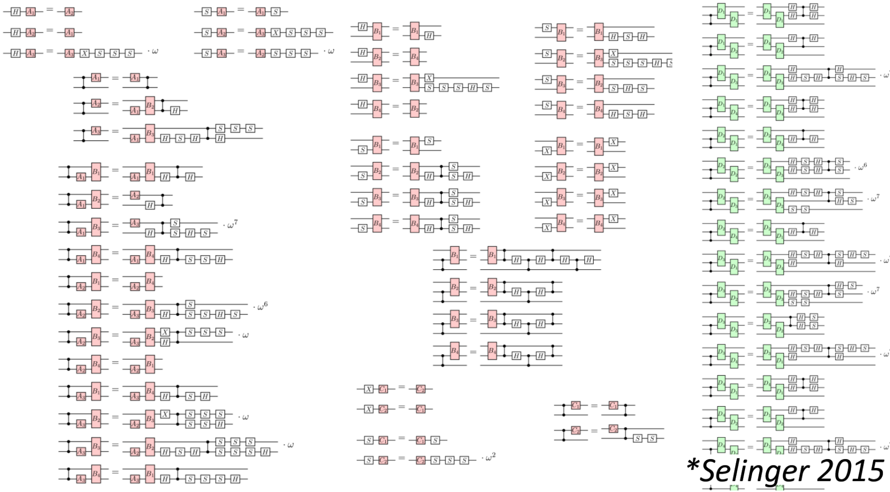


More circuit equalities



*Selinger 2015

And more circuit equalities



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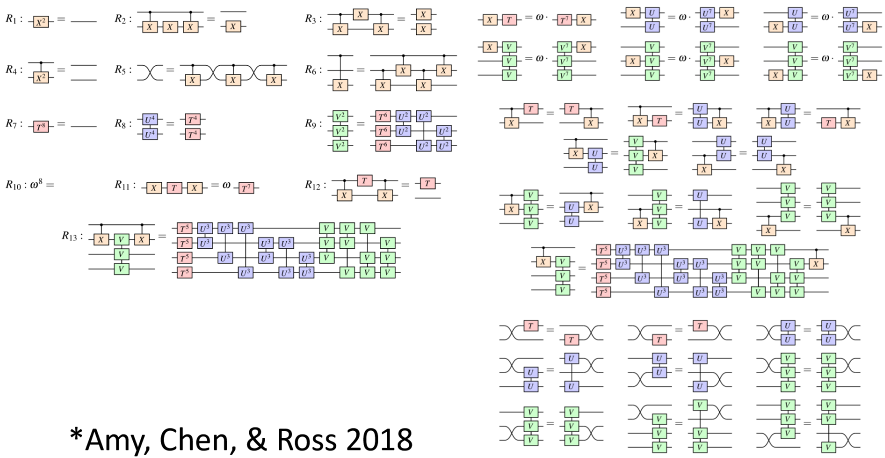
And even more circuit equalities



*Amy, Chen, & Ross 2018

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Enter ZX-diagrams

ZX-diagrams

What gates are to circuits, *spiders* are to ZX-diagrams.

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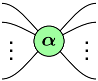
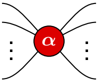
Z-spider

$$|0\cdots 0\rangle\langle 0\cdots 0| + e^{i\alpha}\,|1\cdots 1\rangle\langle 1\cdots 1|$$



X-spider

$$|+\cdots +\rangle\langle +\cdots +| + e^{i\alpha}\,|-\cdots -\rangle\langle -\cdots -|$$



ZX-diagrams

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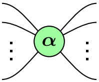
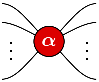
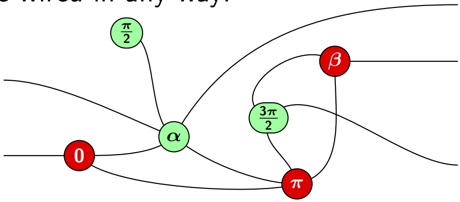
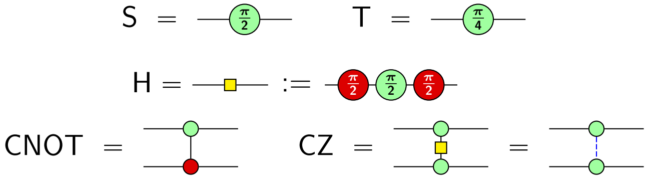
Z-spider	X-spider
$\lvert 0\cdots 0\rangle\langle 0\cdots 0\rvert$ $+e^{i\alpha}\lvert 1\cdots 1\rangle\langle 1\cdots 1\rvert$	$\lvert +\cdots +\rangle\langle +\cdots +\rvert$ $+\,e^{i\alpha}\lvert -\cdots -\rangle\langle -\cdots -\rvert$
	

Spiders can be wired in any way:



Quantum gates as ZX-diagrams

Every quantum gate can be written as a ZX-diagram:



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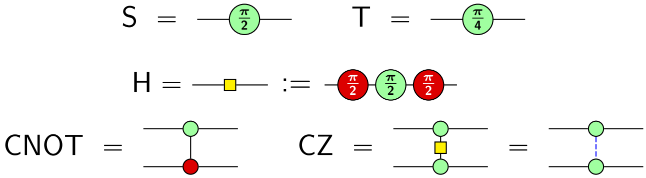
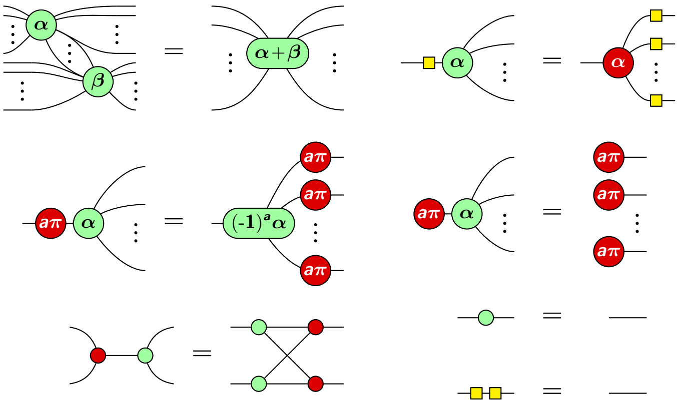


Universality

Any linear map between qubits can be represented as a ZX-diagram.

Rules for ZX-diagrams: The ZX-calculus



$\alpha, \beta \in [0, 2\pi],\ a \in \{0, 1\}$

Completeness of the ZX-calculus

Theorem (Vilmart 2018)

If two ZX-diagrams represent the same linear map, then they can be transformed into one another using the previous rules (and one additional one).

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So instead of dozens of circuit equalities,
we just need a few simple rules.

Optimization using ZX-diagrams

- Write circuit as ZX-diagram.

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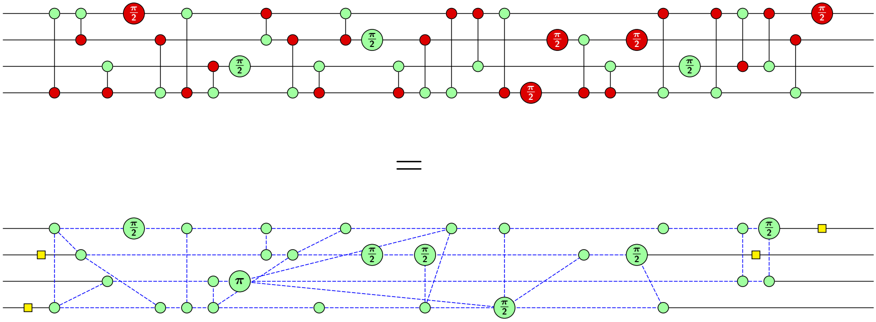
- Simplify the diagram.
- *Extract* a circuit from the diagram.

Graph-like diagrams



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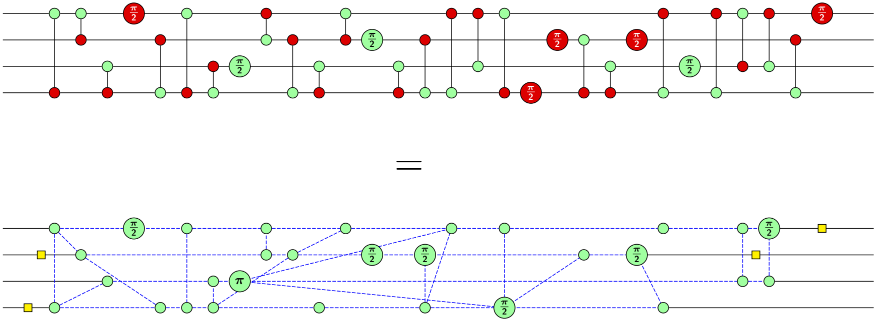
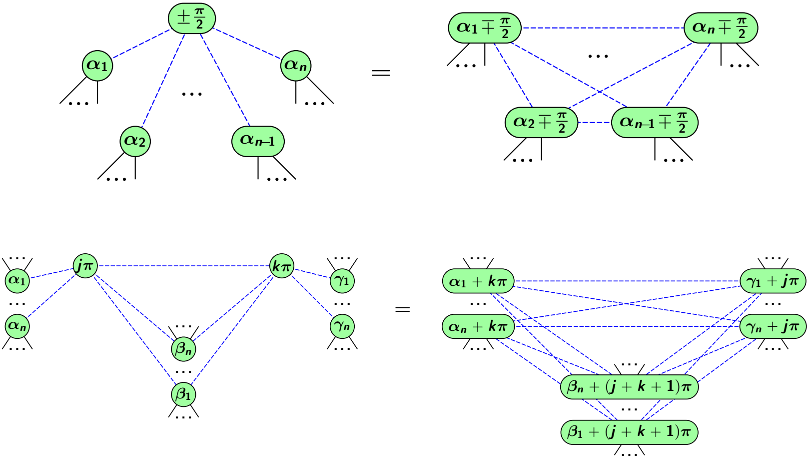


Now we are ready for simplification.

The game: Remove as many interior vertices as possible.

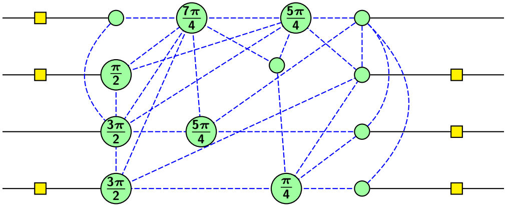
The tools: Local complementation and pivoting



Duncan, Kissinger, Perdrix, vdW (2019)

Circuit extraction

Example result after simplification:



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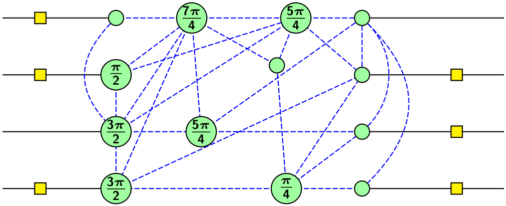

The important thing:
We can turn this back into a circuit.

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- It implements our rewrites and circuit extraction.
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Demonstration time

Future work

- Allow routing for restricted architectures
  (With help from the Unitary Fund)

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- Closer integration with other libraries

Thank you for your attention

Further Reading:

Backens 2014, arXiv:1307.7025
The ZX-calculus is complete for stabilizer quantum mechanics

Vilmart 2018, arXiv:1812.09114
A Near-Optimal Axiomatisation of ZX-Calculus for Pure Qubit Quantum Mechanics

Duncan, Kissinger, Perdrix, vdW 2019, arXiv:1902.03178
Graph-theoretic Simplification of Quantum Circuits with the ZX-calculus