An effect-theoretic reconstruction of quantum theory

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ACT2019
19th of July 2019

Why Quantum Theory?

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Its mathematical description is not particularly compelling:

- Systems are described by C*-algebras.
- States are density matrices.
- Dynamics are completely positive maps.
- Measurement outcomes are governed by the trace rule.
- Composite systems are formed using the tensor product.

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Not clear at all why this describes nature so well.

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Find sensible physical requirements from which it follows.

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If successful, we can say:

Quantum theory describes nature because
"it couldn't have been any other way"

(without nature being that much weirder)

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In this talk:
"Any theory with well-behaved pure maps is quantum theory"
All axioms taken from *effectus theory*

A suitable framework

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An effectus	$\approx$	'generalised generalised probabilistic theory'
real numbers	$\Rightarrow$	effect monoids
vector spaces	$\Rightarrow$	effect algebras.

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2. The maps $v, w : (I+I)+I \to I+I$ given by

$$v = [[\kappa_1, \kappa_2], \kappa_2] \text{ and } w = [[\kappa_2, \kappa_1], \kappa_2] \quad \text{are jointly monic}$$

(i.e. $v \circ f = v \circ g$ and $w \circ f = w \circ g$, then $f = g$).

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  In particular any (causal) general probabilistic theory.
- Opposite category of von Neumann algebras

Basic definitions and consequences

- *Partial maps*: $f : X \to Y + I$.
- *States*: $\mathrm{St}(X) := \mathrm{Hom}(I, X)$.
- *Effects*: $\mathrm{Eff}(X) := \mathrm{Hom}(X, I + I)$.
- *Scalars*: $\mathrm{Hom}(I, I + I)$.

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- The states form an *abstract convex set*.
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Definition of effectus is basically chosen to make these things true

Effect algebras

Definition

An *effect algebra* $(E, 0, 1, +, (\cdot)^{\perp})$ is a set E with partial commutative associate "addition" $+$ and involution $(\cdot)^{\perp}$ such that

- $(x^{\perp})^{\perp} = x$,
- $x + x^{\perp} = 1$,
- If $x + 1$ is defined, then $x = 0$.

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Note: Effect algebra is partially ordered by $x \leqslant y$ iff $\exists z : x + z = y$.

Baby effectus

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A *Effect theory* is a category $\mathbf{B}$ with designated object I such that $\mathrm{Hom}(A, I)$ is an effect algebra, and for any $f : B \to A$: $0 \circ f = 0$, $(p + q) \circ f = (p \circ f) + (q \circ f)$.

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Very basic structure, we need more assumptions!

Compressions and filters

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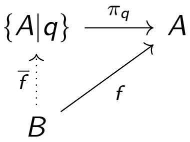

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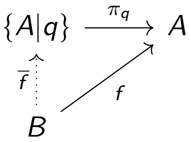

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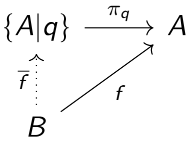
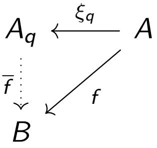
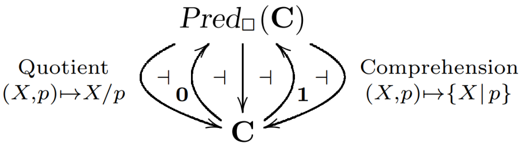


A *filter* for $q : A \to I$ is a map $\xi_q : A \to A_q$ with $1 \circ \xi \leqslant q$, such that for all $f : A \to B$ with $1 \circ f \leqslant q$:



Quotient and Comprehension: All the adjunctions!



$\mathrm{Pred}_{\square}(\mathbf{C})$:
Objects are $(X, p : X \to I)$.
Morphisms: $f : (X, p) \to (Y, q)$ is
$f : X \to Y$ with $p^{\perp} \geqslant q^{\perp} \circ f$.

Source: arXiv:1512.05813, p.97

See also: Cho, Jacobs, Westerbaan[2] 2015. *Quotient–Comprehension Chains*

Example

Let $\mathbf{Mat}_{\mathbb{C}}^{\mathrm{op}}$ be the opposite category of positive sub-unital maps $f : M_n(\mathbb{C}) \to M_m(\mathbb{C})$. I.e $a \geqslant 0 \implies f(a) \geqslant 0$ and $f(1) \leqslant 1$.

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An *effect* then corresponds to $q \in M_n(\mathbb{C})$ with $0 \leqslant q \leqslant 1$.

Write $q = \sum_i \lambda_i q_i$ with $\lambda_i > 0$, $q_i q_j = \delta_{ij} q_i$.
Define $\lceil q \rceil = \sum_i q_i$. $\lfloor q \rfloor = \sum_{i;\lambda_i = 1} q_i$.

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The projection $\pi_q : M_n(\mathbb{C}) \to \lfloor q \rfloor M_n(\mathbb{C}) \lfloor q \rfloor$ is a compression.

$\xi_q : \lceil q \rceil M_n(\mathbb{C}) \lceil q \rceil \to M_n(\mathbb{C})$ with $\xi_q(p) = \sqrt{q} p \sqrt{q}$ is a filter.

Images, kernels and cokernels

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An effect theory has images, and for all sharp effects compressions and filters if and only if the category has all kernels and cokernels.

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In fact: compressions *are* kernels, and filters for sharp effects *are* cokernels.
$\Rightarrow$ filters are "fuzzy" cokernels.

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Remark

From definition it is not clear that pure maps are closed under composition. But: In $\mathbf{Mat}_{\mathbb{C}}^{\mathrm{op}}$ it is true.
Also: there is an obvious dagger on pure maps in $\mathbf{Mat}_{\mathbb{C}}^{\mathrm{op}}$.

Pure effect Theories

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- Category of *real* C*-algebras.
- $\mathbf{EJA}^{\mathrm{op}}_{\mathrm{psu}}$: positive sub-unital maps between *Euclidean Jordan algebras*.

Euclidean Jordan algebras

Definition

A *Euclidean Jordan algebra* (EJA) $(E, \langle \cdot, \cdot \rangle, *, 1)$ is a real Hilbert space with a product that satisfies $\forall a, b, c$:

$$a * 1 = a \quad a * b = b * a \quad a * (b * a^2) = (a * b) * a^2 \quad \langle a * b, c \rangle = \langle b, a * c \rangle$$

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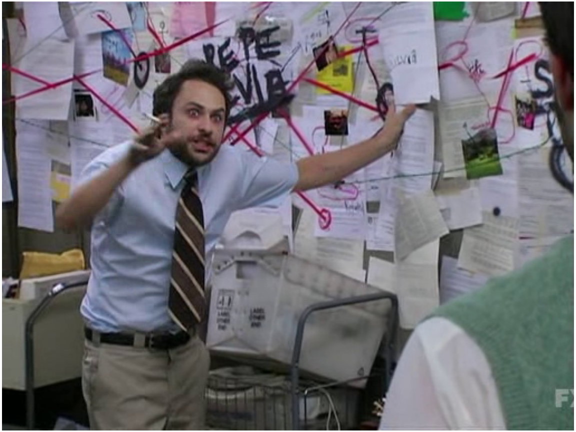
Example: $M_n(F)^{\mathrm{sa}}$ — self-adjoint matrices over $F = \mathbb{R}, \mathbb{C}, \mathbb{H}$ with $A * B := \frac{1}{2}(AB + BA)$ and $\langle A, B\rangle := \mathrm{tr}(AB)$.


HEY EVERYBODY, THIS THING IS A JORDAN ALGEBRA!
SEE? NOBODY CARES

Me explaining why Jordan algebras are cool:



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Operational effect theory $\approx$ generalized probabilistic theory

Main result 1: Everything is a Jordan algebra

Theorem

Let $\mathbf{B}$ be an operational PET. Then there is a functor $F : \mathbf{B} \to \mathbf{EJA}_{psu}^{\mathrm{op}}$ with $F(\mathrm{Eff}(A)) \cong \mathrm{Eff}(F(A))$.

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"Operational PETs are Euclidean Jordan algebras"

Monoidal effect theories

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An effect theory is *monoidal* when it is monoidal with I as unit such that tensor preserves addition. A PET is monoidal if the subcategory of pure maps is in addition also monoidal.

Quantum Theory Reconstructed

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Let $\mathbf{B}$ be a monoidal operational PET. Then there is a functor $F : \mathbf{B} \to \mathbf{C}^{\mathrm{op}}$ with $F(\mathrm{Eff}(A)) \cong \mathrm{Eff}(F(A))$ where $\mathbf{C}$ is the category of real or complex C*-algebras.

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Furthermore, if effects separate maps, then it is faithful and C*-algebras must be complex.

Recall the assumptions:

1. All maps have images.
2. When q is sharp, $q^{\perp}$ is sharp.
3. All effects have filters and compressions.
4. The pure maps form a monoidal dagger-category.
5. If π_q is a compression for sharp q, then $\pi_q^{\dagger}$ is a filter for q.
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