Quantum Circuit Optimisation with the ZX-calculus

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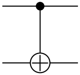
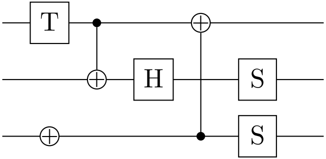
- So quantum circuits should contain as few gates as possible.
- Several important metrics:
  - Gate-depth
  - 2-qubit gate count
  - Number of T gates: *T-count* $[\ \mathrm{T} = R_Z(\frac{\pi}{4})\ ]$

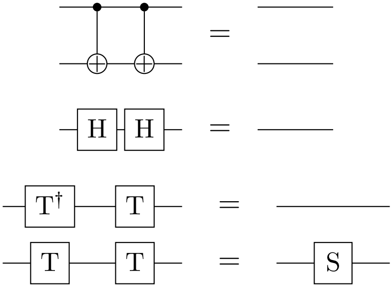
Circuit diagrams

NOT $=$  CNOT $=$ 

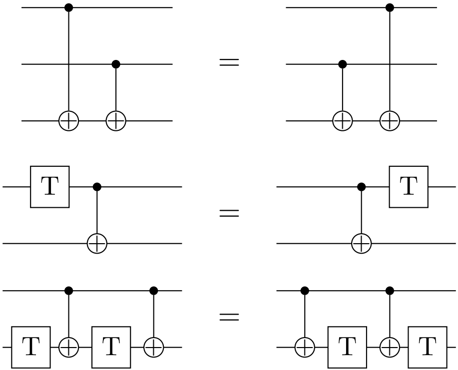
An example quantum circuit:



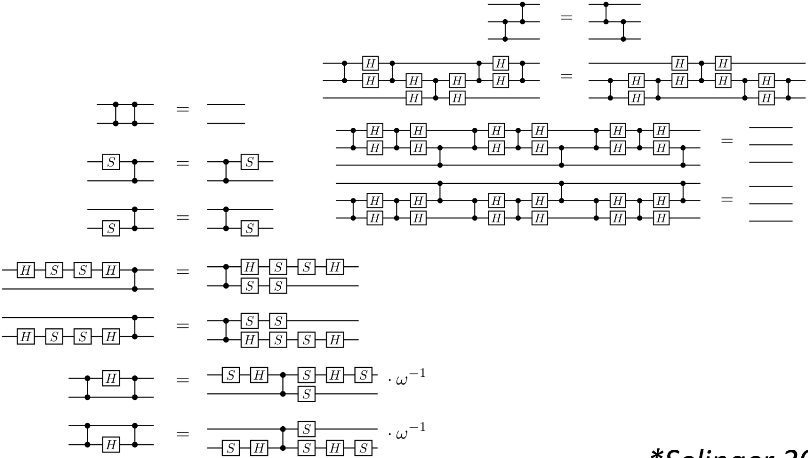
Circuit identities



Gate commutation

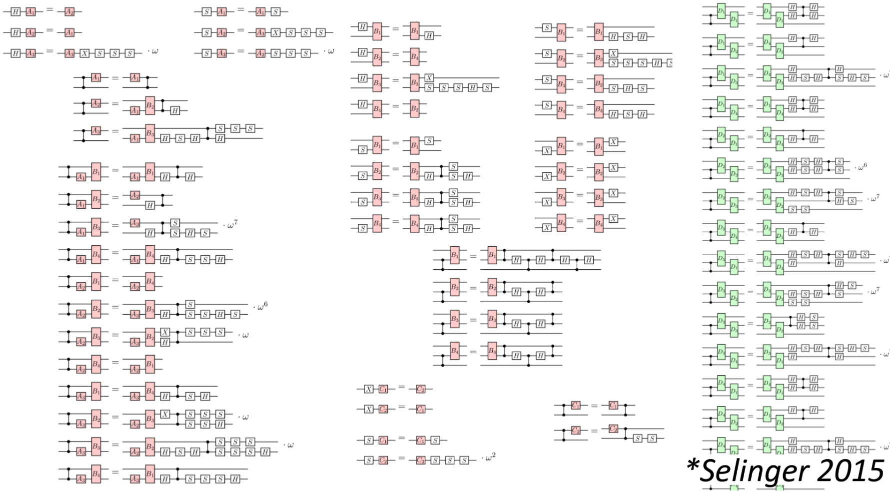


More circuit equalities



*Selinger 2015

And more circuit equalities



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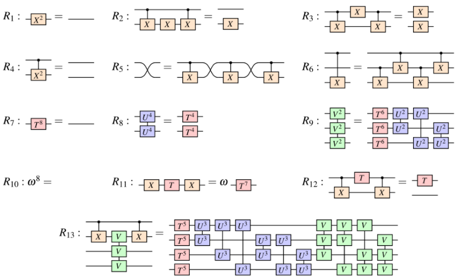

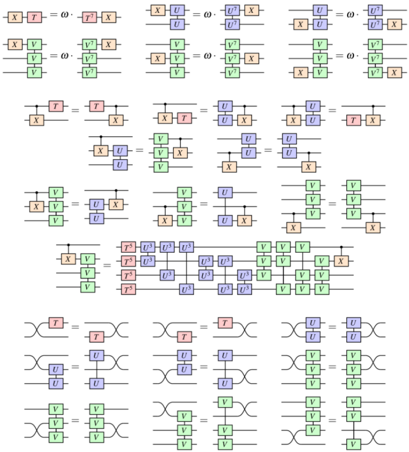

*Amy, Chen, & Ross 2018

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Enter ZX-diagrams

ZX-diagrams

What gates are to circuits, *spiders* are to ZX-diagrams.

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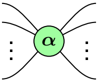
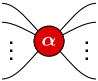
Z-spider

$$|0\cdots0\rangle\langle0\cdots0| + e^{i\alpha}|1\cdots1\rangle\langle1\cdots1|$$



X-spider

$$|+\cdots+\rangle\langle+\cdots+| + e^{i\alpha}|-\cdots-\rangle\langle-\cdots-|$$



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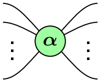

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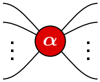
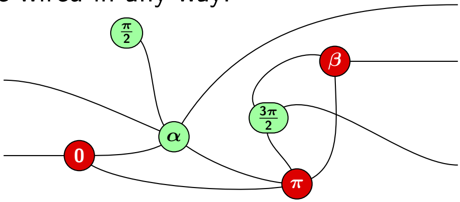
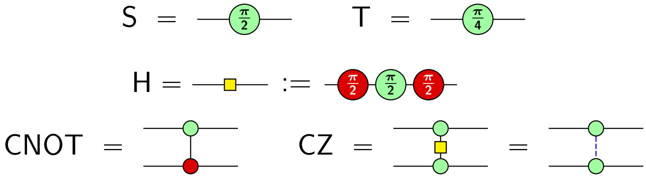


Spiders can be wired in any way:



Quantum gates as ZX-diagrams

Every quantum gate can be written as a ZX-diagram:



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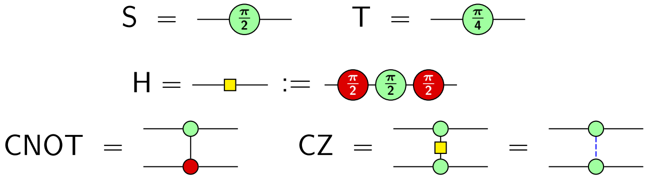
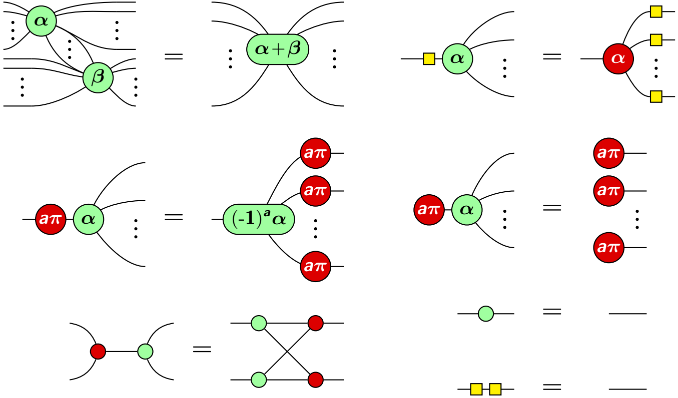


Universality

Any linear map between qubits can be represented as a ZX-diagram.

Rules for ZX-diagrams: The ZX-calculus



$\alpha, \beta \in [0, 2\pi]$, $a \in \{0, 1\}$

Completeness of the ZX-calculus

Theorem (Vilmart 2018)

If two ZX-diagrams represent the same linear map, then they can be transformed into one another using the previous rules (and one additional one).

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So instead of dozens of circuit equalities,
we just need a few simple rules.

Optimisation using ZX-diagrams

- Write circuit as ZX-diagram.

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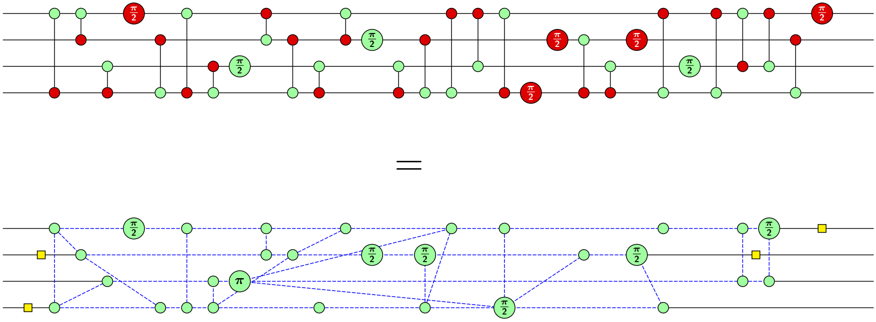
- Simplify the diagram.
- *Extract* a circuit from the diagram.

PyZX

- PyZX is an open-source Python library.
- `https://github.com/Quantomatic/pyzx`
- It allows easy manipulation of large ZX-diagrams.

Graph-like diagrams



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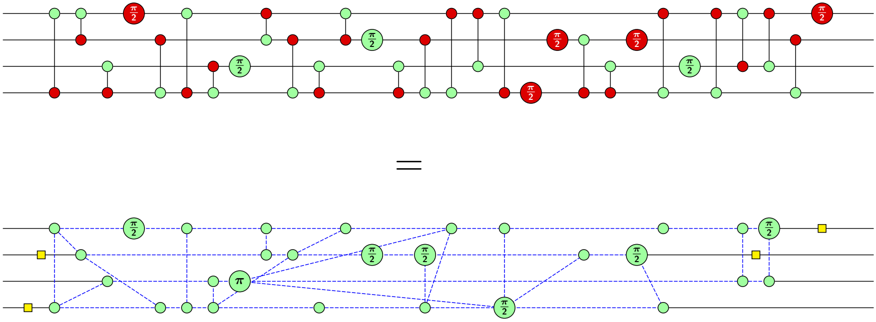
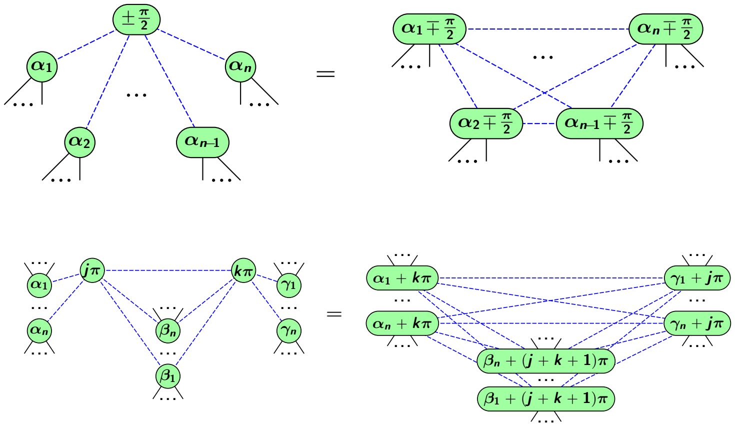


Now we are ready for simplification.
The game: Remove as many interior vertices as possible.

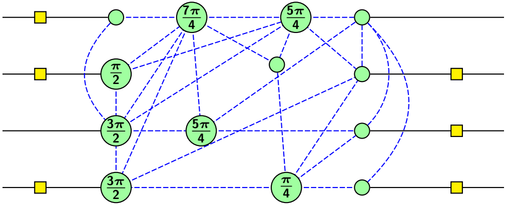
The tools: Local complementation and pivoting



Duncan, Kissinger, Perdrix, vdW (2019)

Example

Example result after simplification:



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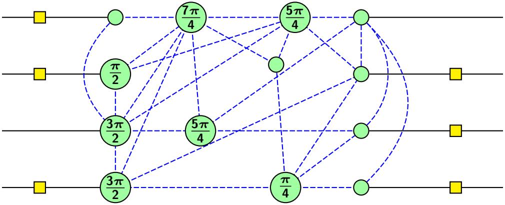

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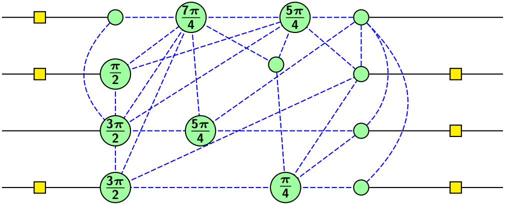
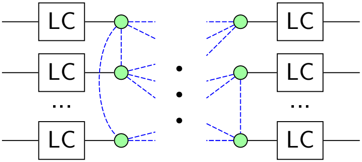


Problem: does not look a circuit.
Solution: all rewrites preserve *gflow*.

- Duncan, Perdrix, Kissinger, vdW (2019). *Graph-theoretic Simplification of Quantum Circuits with the ZX-calculus*.
- Backens, Miller-Bakewell, de Felice, Lobski, vdW (2020). *There and back again: A circuit extraction tale*.

Clifford simplification

Clifford circuits are reduced to a pseudo-normal form:



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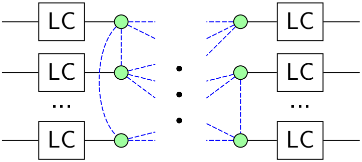
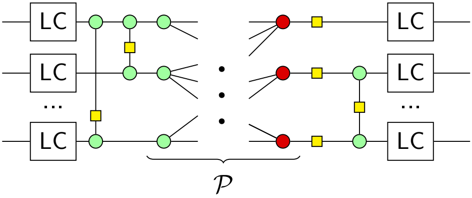
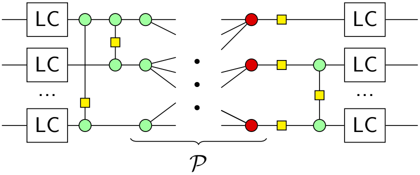


This is equal to:



e.g. $\mathcal{P}\,|x_1, x_2, x_3, x_4\rangle \mapsto |x_1 \oplus x_2, x_1 \oplus x_3, x_4, x_3\rangle\,.$

Clifford normal form



- Extracts to circuit with 8 layers:
- H - S - CZ - CNOT - H - CZ - S - H

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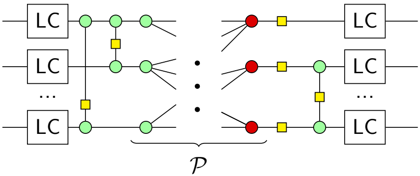

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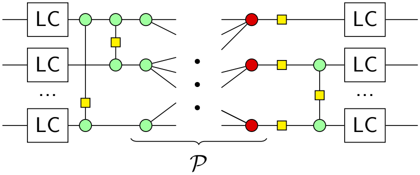

- Extracts to circuit with 8 layers:
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- Asymptotically optimal number of free parameters, like normal form of [Maslov & Roetteler 2019].
- But additionally, linear nearest neighbour depth of $9n - 2$, a new record (Recently matched by [Bravyi & Maslov 2020]).

Non-Clifford optimisation

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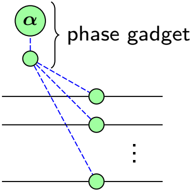
Additional rules for *phase gadgets*:



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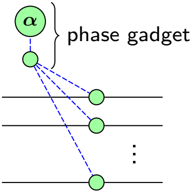

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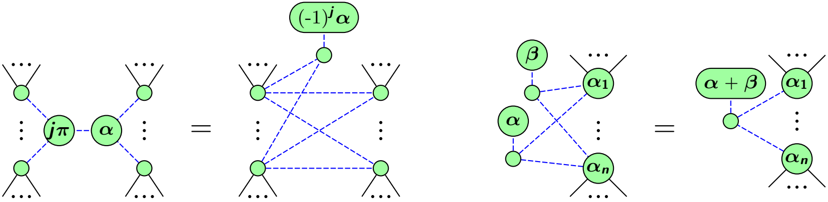

Kissinger, vdW 2019: *Reducing T-count with the ZX-calculus*

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- At time of publishing, our method improved upon previous best T-counts for 6/36 benchmark circuits — in one case by 50%.
- Combining with TODD [Heyfron & Campbell 2018] we improved T-counts for 20/36 circuits.
- Note: [Zhang & Chen 2019] use a different method that achieves nearly identical T-counts.

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- Improves on previous best for quantum chemistry circuits of [Cowtan et al. 2019].

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Using the ZX-calculus we found new techniques to improve depth, two-qubit gate count and T-count of realistic benchmark circuits.

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Future work:

- Allow routing for restricted architectures.
- Improve extraction to reduce CNOT count.
- Find ways to incorporate ancillae.

Thank you for your attention

References

Duncan, Kissinger, Perdrix & vdW 2019, arXiv:1902.03178
Graph-theoretic Simplification of Quantum Circuits with the ZX-calculus

Kissinger & vdW 2019, arXiv:1903.10477
Reducing T-count with the ZX-calculus

Backens, Miller-Bakewell, de Felice, Lobski & vdW 2020, arXiv:2003.01664
There and back again: A circuit extraction tale

Heyfron & Campbell 2018, arxiv:1712.01557
An Efficient Quantum Compiler that reduces T count

Maslov & Roetteler 2018, arXiv:1705.09176
Shorter stabilizer circuits via Bruhat decomposition and quantum circuit transformations

Zhang & Chen 2019, arXiv:1903.12456
Optimizing T gates in Clifford+T circuit as $\pi/4$ rotations around Paulis

Bravyi & Maslov 2020, arXiv:2003.09412
Hadamard-free circuits expose the structure of the Clifford group