

# Quantum circuit optimisation, verification, and simulation with PyZX

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PyZX: a Python library for manipulating large ZX-diagrams



# Quantum computation

- ▶ Quantum computation is done by *quantum circuits*.

# Quantum computation

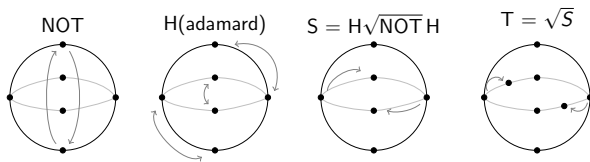
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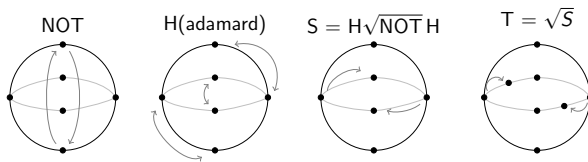
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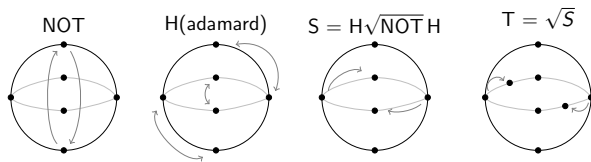
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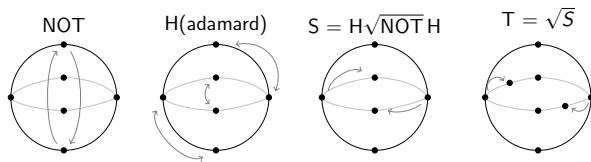


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
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- ▶ These are all the gates you need.
- ▶ Our objective (for now) is to minimize number of gates needed

# Circuit diagrams

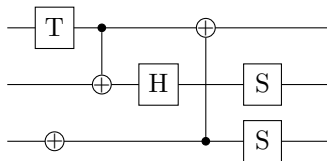
NOT = 

CNOT = 

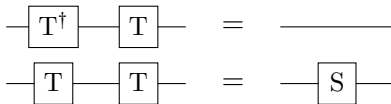
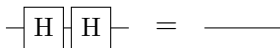
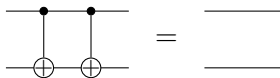
# Circuit diagrams

$$\text{NOT} = \text{---}\oplus\text{---} \qquad \text{CNOT} = \begin{array}{c} \text{---}\bullet\text{---} \\ | \\ \text{---}\oplus\text{---} \end{array}$$

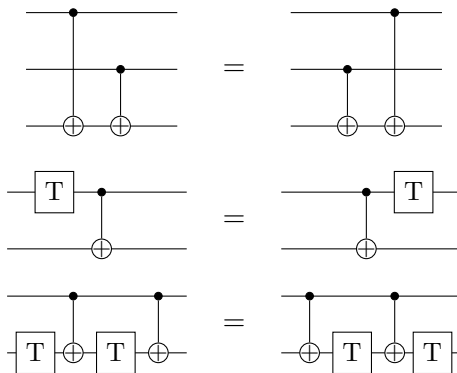
An example quantum circuit:



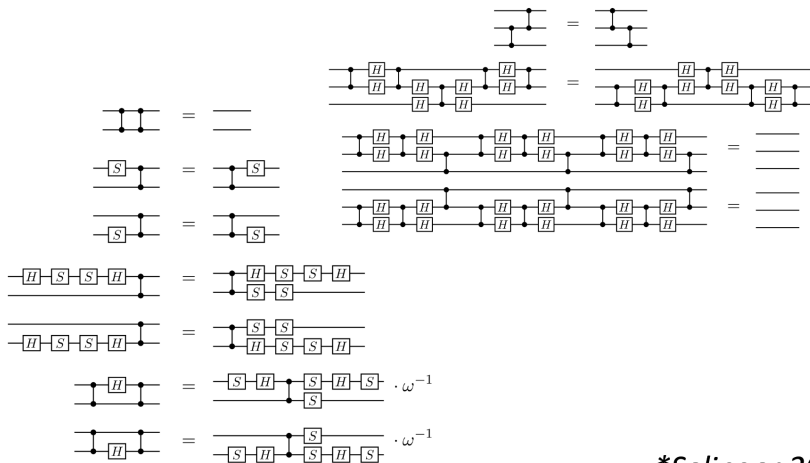
# Circuit identities



# Gate commutation

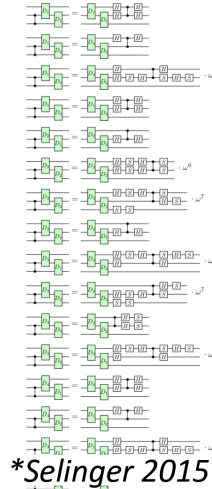


# More circuit equalities



*\*Selinger 2015*

# And more circuit equalities



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# And even more circuit equalities

$$R_1: \boxed{X^2} = \text{---}$$

$$R_2: \begin{array}{c} \text{---} \\ | \\ \boxed{X} \text{---} \boxed{X} \text{---} \boxed{X} \\ | \\ \text{---} \end{array} = \boxed{X}$$

$$R_3: \begin{array}{c} \boxed{X} \\ | \\ \boxed{X} \text{---} \boxed{X} \end{array} = \boxed{X}$$

$$R_4: \boxed{X^2} = \text{---}$$

$$R_5: \text{---} \times \text{---} = \begin{array}{c} \text{---} \\ | \\ \boxed{X} \text{---} \boxed{X} \text{---} \boxed{X} \\ | \\ \text{---} \end{array}$$

$$R_6: \begin{array}{c} \text{---} \\ | \\ \boxed{X} \end{array} = \begin{array}{c} \boxed{X} \\ | \\ \boxed{X} \text{---} \boxed{X} \end{array}$$

$$R_7: \boxed{T^4} = \text{---}$$

$$R_8: \begin{array}{c} \boxed{U^2} \\ | \\ \boxed{U^2} \end{array} = \begin{array}{c} \boxed{T^4} \\ | \\ \boxed{T^4} \end{array}$$

$$R_9: \begin{array}{c} \boxed{V^2} \\ | \\ \boxed{V^2} \end{array} = \begin{array}{c} \boxed{T^6} \boxed{U^2} \boxed{U^2} \\ | \\ \boxed{T^6} \boxed{U^2} \boxed{U^2} \end{array}$$

$$R_{10}: \omega^8 =$$

$$R_{11}: \boxed{X} \boxed{T} \boxed{X} = \omega \cdot \boxed{T^2}$$

$$R_{12}: \begin{array}{c} \boxed{T} \\ | \\ \boxed{X} \text{---} \boxed{X} \end{array} = \boxed{T}$$

$$R_{13}: \begin{array}{c} \boxed{X} \boxed{V} \boxed{X} \\ | \\ \boxed{V} \\ | \\ \boxed{V} \end{array} = \begin{array}{c} \boxed{T^2} \boxed{U^2} \boxed{U^2} \boxed{U^2} \\ | \\ \boxed{T^2} \boxed{U^2} \boxed{U^2} \boxed{U^2} \end{array}$$

$$\boxed{X} \boxed{T} = \omega \cdot \boxed{T^2} \boxed{X}$$

$$\boxed{X} \boxed{U} = \omega \cdot \boxed{U^2} \boxed{X}$$

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$$\begin{array}{c} \boxed{X} \boxed{V} \\ | \\ \boxed{V} \end{array} = \begin{array}{c} \boxed{T^2} \boxed{U^2} \boxed{U^2} \boxed{U^2} \\ | \\ \boxed{T^2} \boxed{U^2} \boxed{U^2} \boxed{U^2} \end{array}$$

$$\boxed{T} = \boxed{T}$$

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\*Amy, Chen, & Ross 2018



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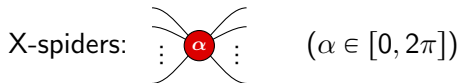
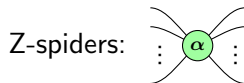
Enter ZX-diagrams

# ZX-diagrams

What gates are to circuits,  
*spiders* are to ZX-diagrams.

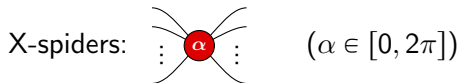
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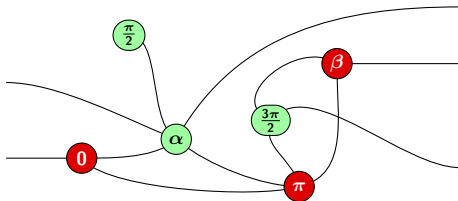


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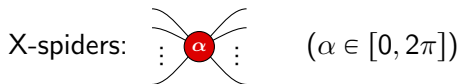


Spiders can be wired in any way:

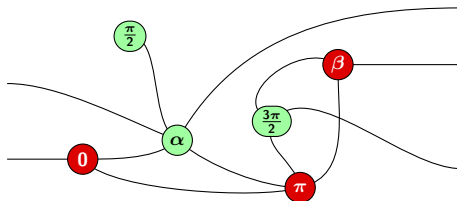


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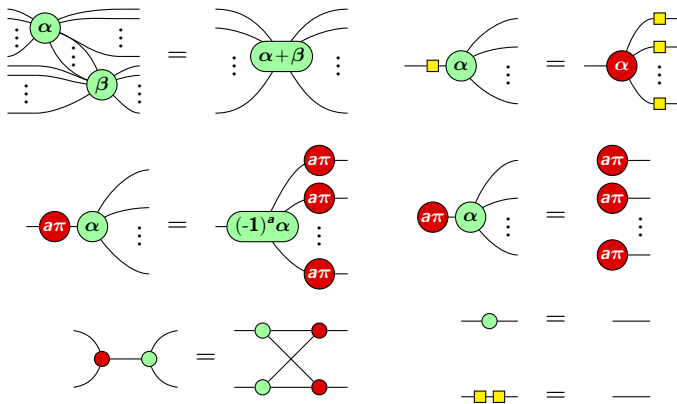
Note: “Only connectivity matters”







# Rules for ZX-diagrams: The ZX-calculus



$$\alpha, \beta \in [0, 2\pi], a \in \{0, 1\}$$

# Completeness of the ZX-calculus

## Theorem

If two ZX-diagrams represent the same computation, then they can be transformed into one another using the previous rules (and one additional one).

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So instead of dozens of circuit equalities, we just have a few simple rules.

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- ▶ Does circuit optimisation
- ▶ Does circuit verification
- ▶ Does circuit simulation (WIP)

# Demonstration time

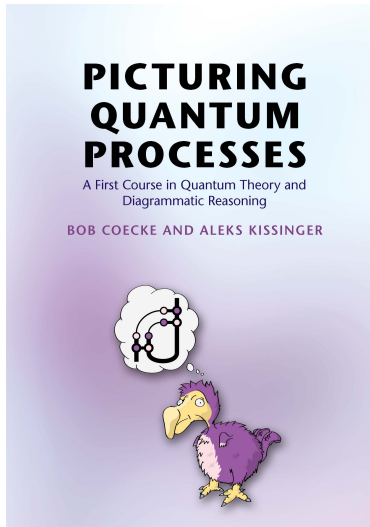
# Want to learn more?

- [github.com/Quantomatic/pyzx](https://github.com/Quantomatic/pyzx)
- [zxcalculus.com](https://zxcalculus.com)



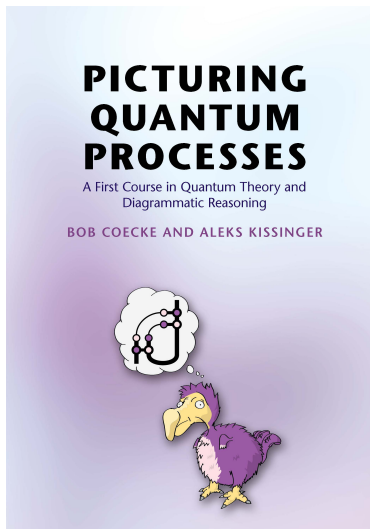
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Thank you  
for your attention!