Universal measurement-based quantum computation with Mølmer-Sørensen interactions and just two measurement bases

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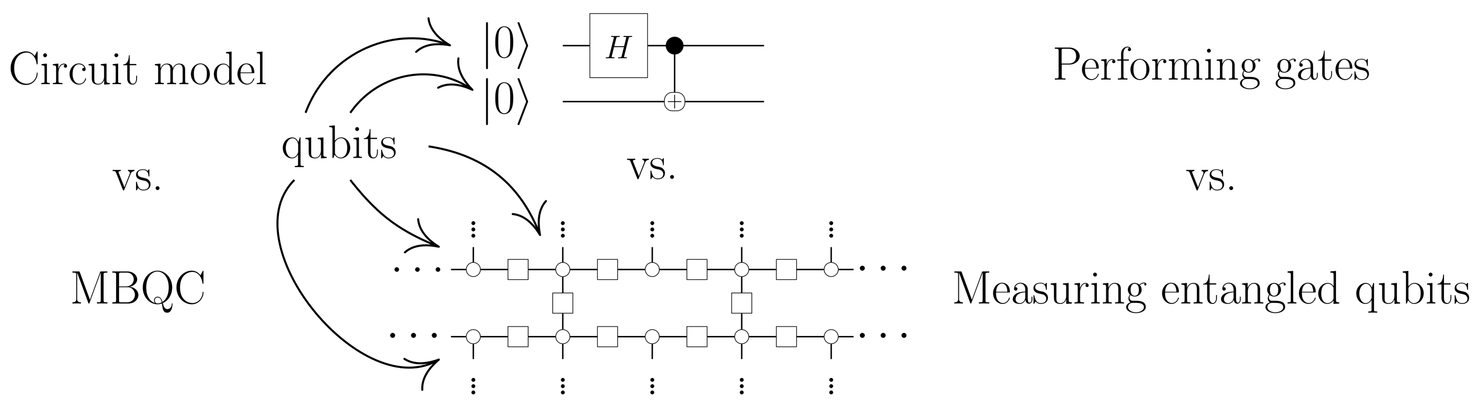
Measurement-based quantum computation



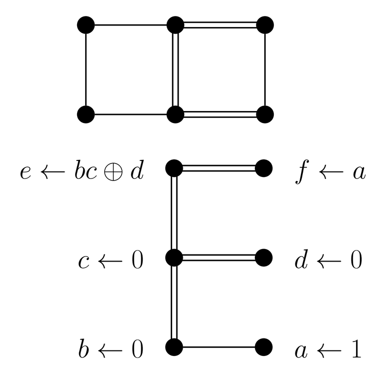
- ⋄ Start with a highly entangled resource state.
- ⋄ Perform some measurements.
- ⋄ Perform more measurements depending on previous outcomes (*feed-forward*).
- ⋄ Most common model: *The one-way model*. The resource state is a stabiliser *graph state* and non-Pauli measurements introduce the universality of computation.
- ⋄ Graph states: a stabiliser state which can be described by an undirected graph, where each edge represents a pair of qubits entangled via a controlled-Z operation as shown above.
- ⋄ Stabiliser computation is efficiently classically simulable, so universality comes from non-stabiliser measurements.

Mølmer-Sørensen Interactions

- ⋄ MS-interactions are a common way to implement an entangling 2-qubit unitary in ion trap systems.
- ⋄ The logical unitary has the form $M_x(\alpha) = \exp(-i\frac{\alpha}{2} X \otimes X)$ (up to a global basis change $X \leftrightarrow Z$).
- ⋄ This logical unitary also appears in transmon-based superconducting qubit systems.
- ⋄ Our model uses these MS-gates to build a non-stabiliser resource state.
- ⋄ We show that: MS-graph state + Pauli-Measurements = Universal computation.

Our model

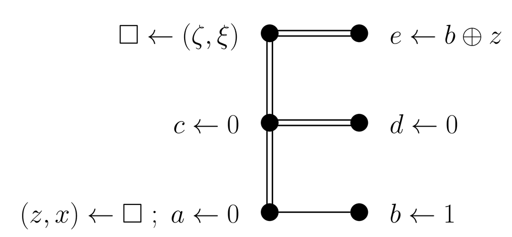
Our graph states consists of nodes connected by either a single or double edge:



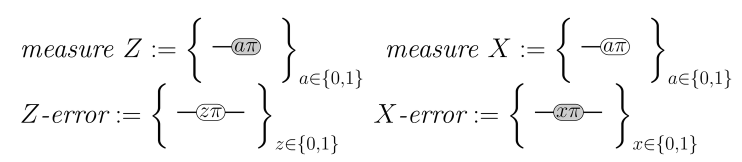
Every node is labelled by a measurement expression like "$c \leftarrow 0$" which determines whether it is to be measured in the Z or X basis. Now:

1. For each vertex initialize a qubit in $|0\rangle$.
2. Apply $M_x(\frac{\pi}{4})$ to every pair of qubits connected by a single edge and $M_x(\frac{\pi}{2}) = M_x(\frac{\pi}{4})^2$ to every pair connected by a double edge.
3. For a qubit labelled "$b \leftarrow \phi(a_1, \ldots, a_n)$", where the values $a_1, \ldots, a_n$ are known, measure in the Z-basis if $\phi(a_1, \ldots, a_n) = 0$ and the X-basis otherwise. In either case, store the measurement result in b.
4. Optionally, perform some classical post-processing on the measurement results.

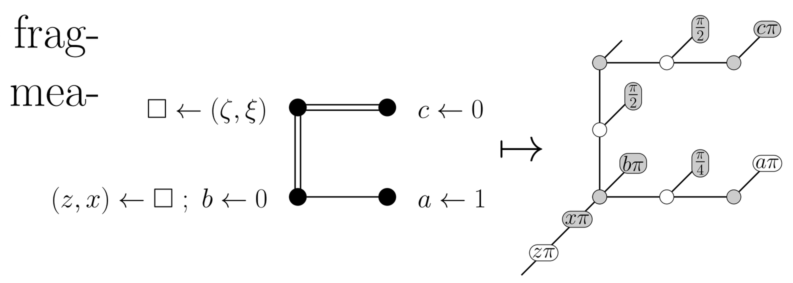
These graph states are built up from pattern fragments. Here "$(z, x) \leftarrow \square$" denotes an input while "$\square \leftarrow (\zeta, \xi)$" denotes an output. (z, x) is whether a Z or X error is propagated into the fragment while (ζ, ξ) are outgoing errors.



We can translate to the ZX calculus by making the identifications on the right.

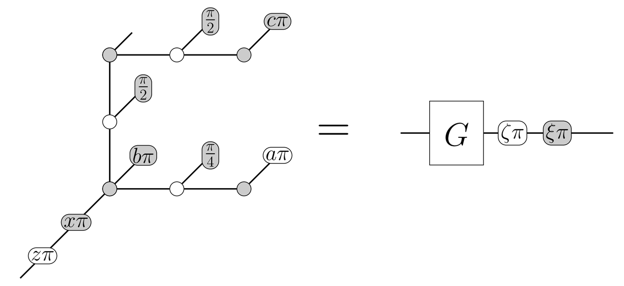


This notation enables us to write fragments by allowing us to deal with measurements and in- and outputs:

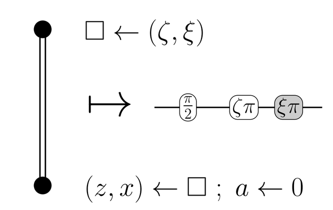


Proving Universality

For universality we need to show that pattern fragments implement certain gates G. E.g. that there exists a reduction in the ZX-calculus:

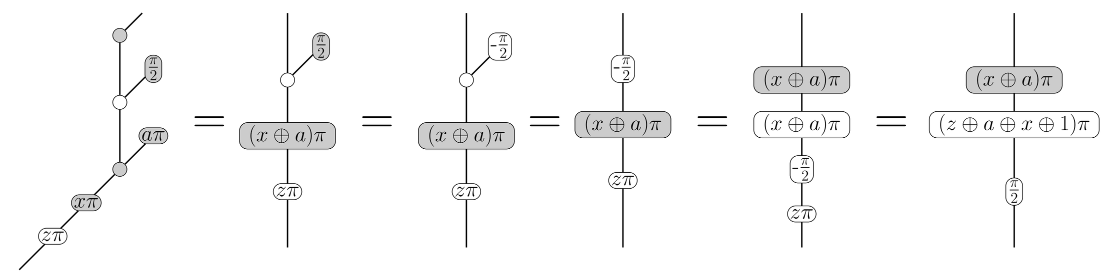


For instance, the following fragment implements a $S := Z(\pi/2)$ gate:

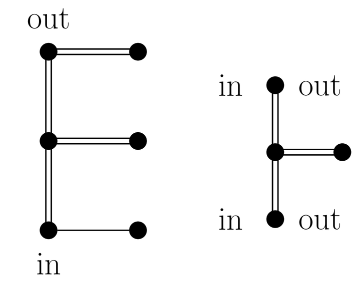


where
$$\begin{cases} \zeta = z \oplus a \oplus x \oplus 1 \\ \xi = x \oplus a \end{cases}$$

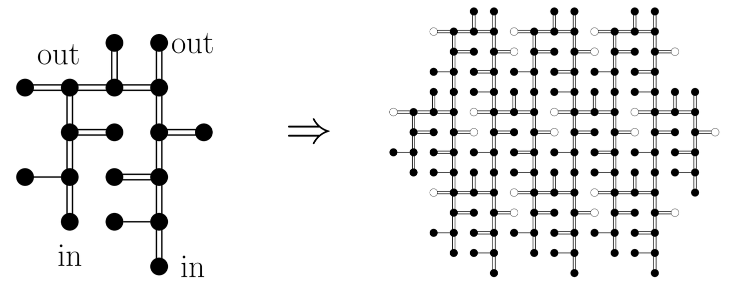
It's correctness can be demonstrated by a series of reductions:



A couple of versatile fragments are this 'E'-shape that depending on the measurements can implement an S, T or H gate and this fragment with 2 inputs and outputs (top and bottom) that can implement either a S⊗S gate or a controlled-Z.

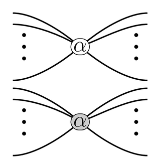


These can in turn be combined into a universal 'LEGO'-like piece that connects in such a way to form a grid allowing universal computation



ZX-notation

We will perform our calculations in *ZX-notation*. This consists of special linear maps from qubits to qubits called Z- and X-*spiders*:



$$:= \quad |0\cdots0\rangle\langle 0\cdots 0| + e^{i\alpha}|1\cdots1\rangle\langle1\cdots1|$$

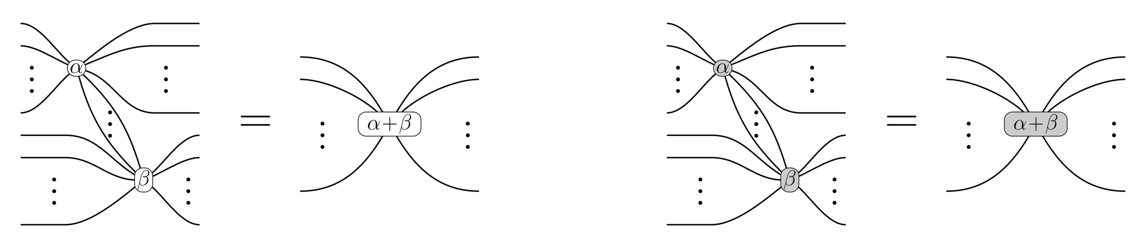
$$:= \quad |+\cdots+\rangle\langle+\cdots+| + e^{i\alpha}|-\cdots-\rangle\langle-\cdots-|$$

A few things about these spiders:

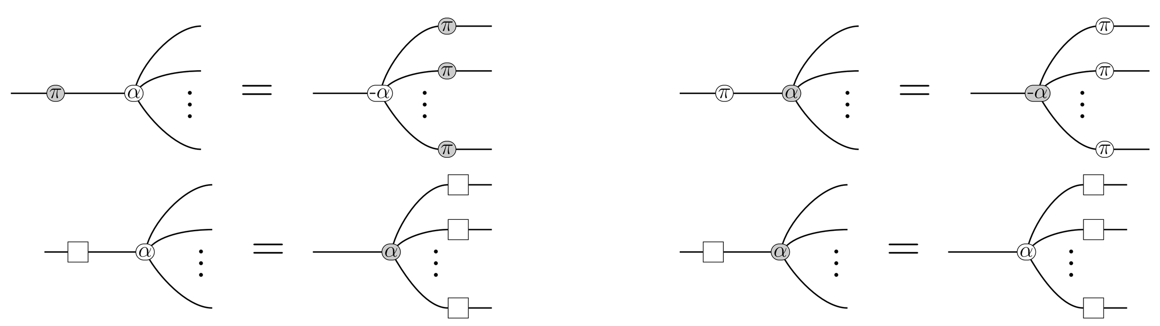
- ⋄ $|0\rangle$ = and $|1\rangle$ = = while $|+\rangle$ = and $|-\rangle$ = =
- ⋄ The standard Pauli gates are $Z(\alpha)$ = and $X(\alpha)$ =
- ⋄ A Hadamard gate is a particular sequence of phase gates: =
- ⋄ The CNOT gate is depicted as two connected spiders:
- ⋄ Two ZX-diagrams are equal when they are equal as undirected graphs

The power of this notation comes from a set of graphical rewrite rules.

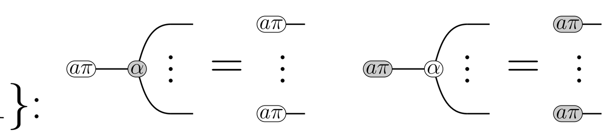
For instance, two spiders of the same colour fuse together:



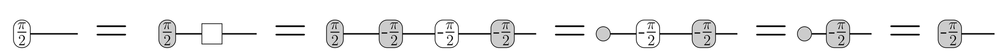
Additionally: Pauli X and Z gates can be *pushed trough* spiders of the opposite colour whichs flips the phase. Hadamard gates change the colour of the spider:



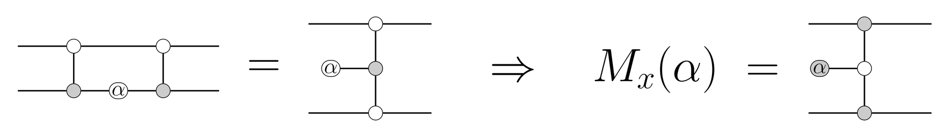
Some states are *copied* by spiders: when $a \in \{0, 1\}$:



A small example of how the rewrite rules can prove nontrivial equalities:



MS-gate in ZX-notation: First note that $M_z(\alpha) = \exp(-i\frac{\alpha}{2} Z \otimes Z)$ which is proportional to CNOT$(I \otimes Z(\alpha))$CNOT, so



because $M_x(\alpha)$ is just a colour change away from $M_z(\alpha)$.
For the special value of $\alpha = \frac{\pi}{2}$ we can go a step further: