An effect-theoretic reconstruction of quantum theory

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In this talk

- The use of physical principles in physics
- A brief history of (reconstructing) quantum theory
- Generalised probabilistic theories
- Effectus theory and a new reconstruction

Why Quantum Theory?

Why Relativity?

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- At the time there wasn't much evidence supporting this.
- It took him 10 years to formalise his third principle:
- Gravitational and inertial acceleration are equivalent.
- Incredibly, his theory still seems correct for large scale structures.

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  (we can test the constancy of the speed of light)
- Aesthetically pleasing.
  (reduces 'why relativity?' to 'why these principles?')
- Helps the search for generalisations
  (because you know you need to break one of these principles)

Back to quantum theory

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- 1932: von Neumann, *Mathematische Grundlagen der Quantenmechanik*.

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Basically, we now still use the mathematical framework specified by von Neumann.

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- The Hilbert space of a composite system is given by the tensor product of the component Hilbert spaces.

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- Modern work (2000-2020) focuses more on *operational frameworks*.

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- In relativity: clocks, rods, events, observers.
- Entropy is a priori an abstract quantity, but via Shannon information theory can be given an operational interpretation.
- Measurement probabilities are operational: 'prepare this state, apply this transformation, do this measurement. Repeat many times and record the probability of observing a certain outcome'.

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- The probability that the outcome associated to a_j is observed when system is in state ω is denoted by $\omega(a_j) \in [0, 1]$, and we have $\sum_j \omega(a_j) = 1$.

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- We have a special 'empty system' I.
- States can then be seen as transformations $\omega : I \to A$, i.e. 'create something from nothing'.
- An effect is a transformation $a : A \to I$, i.e. 'destroy the system'.
- Probabilities are transformations $p : I \to I$.

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- Similarly define $pa_1 + (1-p)a_2$ for effects $a_1, a_2 \in \mathrm{Eff}(A)$. This makes $\mathrm{Eff}(A)$ a convex set.
- We have $\omega(pa_1 + (1-p)a_2) = p\omega(a_1) + (1-p)\omega(a_2)$.

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- Composite systems given by tensor product of matrices.
- Transformations are completely positive trace-non-increasing maps (or equivalently, CP subunital maps in the opposite direction).

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Underlying claim: GPTs can represent any physical theory.

GPTs already assume as given the classical probabilistic framework, and that probabilities are given by real numbers.

This is categorically not very natural.

A suitable categorical framework

Effectus theory

- K. Cho, B. Jacobs, B. Westerbaan & A. Westerbaan (2015): *Introduction to effectus theory*.
- B. Westerbaan (2018): *Dagger and Dilation in the Category of Von Neumann algebras*.
- K. Cho (2019): *Effectuses in Categorical Quantum Foundations*.

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An effectus	$\approx$	'generalised generalised probabilistic theory'
real numbers	$\Rightarrow$	effect monoids
convex spaces	$\Rightarrow$	effect algebras.

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X + Y & \xrightarrow{\text{id}+!} & X + I \\
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\qquad
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2. The maps $v, w : (I+I)+I \to I+I$ given by

$$v = [[\kappa_1, \kappa_2], \kappa_2] \text{ and } w = [[\kappa_2, \kappa_1], \kappa_2] \text{ are jointly monic}$$

(i.e. if $v \circ f = v \circ g$ and $w \circ f = w \circ g$, then $f = g$).

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- Opposite of category of *order unit spaces*
  In particular any (causal) general probabilistic theory.
- Opposite category of von Neumann algebras

Basic definitions and consequences

- *Partial maps*: $f : X \to Y + I$.
- *States*: $\mathrm{St}(X) := \mathrm{Hom}(I, X)$.
- *Effects*: $\mathrm{Eff}(X) := \mathrm{Hom}(X, I + I)$.
- *Scalars*: $\mathrm{Hom}(I, I + I)$.

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- The states form an *abstract convex set*.
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Definition of effectus is basically chosen to make these things true

Effect algebras

Definition

An *effect algebra* $(E, 0, 1, +, (\cdot)^\perp)$ is a set E with partial commutative associative "addition" $+$ and involution $(\cdot)^\perp$ such that

- $(x^\perp)^\perp = x$,
- $x + x^\perp = 1$,
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Note1: Effect algebra is partially ordered by $x \leqslant y$ iff $\exists z : x + z = y$.
Note2: Effect algebras are Eilenberg-Moore algebras of free-forgetful adjunction between bounded posets and orthomodular posets.

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This is what we replace GPTs with. Now we introduce the additional assumptions.

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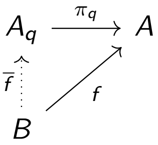

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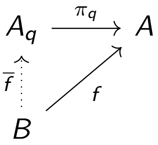

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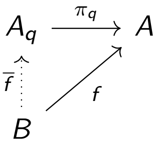
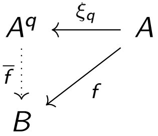
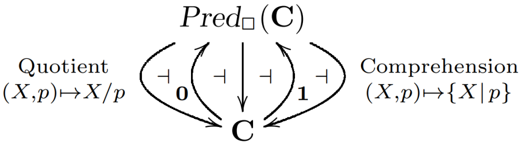


A *filter* for $q : A \to I$ is a map $\xi_q : A \to A^q$ with $1 \circ \xi \leqslant q$, such that for all $f : A \to B$ with $1 \circ f \leqslant q$:



Quotient and Comprehension: All the adjunctions!



$\mathrm{Pred}_{\square}(\mathbf{C})$:
Objects are $(X, p : X \to I)$.
Morphisms: $f : (X, p) \to (Y, q)$ is
$f : X \to Y$ with $p^{\perp} \geqslant q^{\perp} \circ f$.

Source: arXiv:1512.05813, p.97

See also: Cho, Jacobs, Westerbaan^2 2015. *Quotient–Comprehension Chains*

Example

Let $\mathbf{Mat}_{\mathbb{C}}^{\mathrm{op}}$ be the opposite category of positive sub-unital maps $f : M_n(\mathbb{C}) \to M_m(\mathbb{C})$. I.e $a \geqslant 0 \implies f(a) \geqslant 0$ and $f(1) \leqslant 1$.

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An *effect* then corresponds to $q \in M_n(\mathbb{C})$ with $0 \leqslant q \leqslant 1$.

Write $q = \sum_i \lambda_i q_i$ with $\lambda_i > 0$, $q_i q_j = \delta_{ij} q_i$.
Define $\lceil q \rceil = \sum_i q_i$. $\lfloor q \rfloor = \sum_{i;\lambda_i = 1} q_i$.

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The projection $\pi_q : M_n(\mathbb{C}) \to \lfloor q \rfloor M_n(\mathbb{C}) \lfloor q \rfloor$ is a compression.

$\xi_q : \lceil q \rceil M_n(\mathbb{C}) \lceil q \rceil \to M_n(\mathbb{C})$ with $\xi_q(p) = \sqrt{q} p \sqrt{q}$ is a filter.

NOTE: Being universal objects, compressions and filters are unique up to isomorphism.

Images, kernels and cokernels

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An effect theory has images, and for all sharp effects compressions and filters if and only if the category has all kernels and cokernels.

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In fact: compressions *are* kernels, and filters for sharp effects *are* cokernels.
$\Rightarrow$ filters are "fuzzy" cokernels.

Pure maps

Definition

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Also: there is an obvious dagger on pure maps in $\mathbf{Mat}_{\mathbb{C}}^{\mathrm{op}}$.

Some motivation for compression and filter axioms

- A compression relates the subsystem where an effect is certainly true to the original system.
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Some motivation for compression and filter axioms

- A compression relates the subsystem where an effect is certainly true to the original system.
- Conversely, a filter *filters* a subsystem to make an effect true.
- Hence, ‘reversing’ a filter we get a compression and vice versa.
- Note also that if we compose a compression with a filter for the same effect, that we arrive back at the same system.

Pure effect Theories

Definition

A *pure effect theory* (PET) is an effect theory satisfying the following:

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- Category of finite-dimensional *real* C*-algebras.
- $\mathbf{EJA}^{\mathrm{op}}_{\mathrm{psu}}$: positive sub-unital maps between *Euclidean Jordan algebras*.

Euclidean Jordan algebras

Definition

A *Euclidean Jordan algebra* (EJA) $(E, \langle \cdot, \cdot \rangle, *, 1)$ is a real Hilbert space with a commutative unital product that satisfies $\forall a, b, c$:

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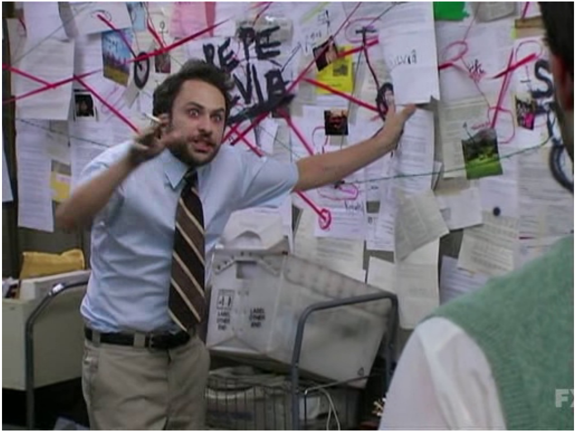
Example: $M_n(F)^{\mathrm{sa}}$ — self-adjoint matrices over $F = \mathbb{R}, \mathbb{C}, \mathbb{H}$ with $A * B := \frac{1}{2}(AB + BA)$ and $\langle A, B \rangle := \mathrm{tr}(AB)$.


HEY EVERYBODY, THIS THING IS A JORDAN ALGEBRA!
SEE? NOBODY CARES

Me explaining why Jordan algebras are cool:



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Operational effect theory $\approx$ generalized probabilistic theory

Main result 1: Everything is a Jordan algebra

Theorem

Let **C** be an operational PET. Then there is a functor $F : \mathbf{C} \to \mathbf{EJA}_{psu}^{\mathrm{op}}$ with $F(\mathrm{Eff}(A)) \cong \mathrm{Eff}(F(A))$.

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"Operational PETs consist of Euclidean Jordan algebras"

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Definition

An effect theory is *monoidal* when it is monoidal, I is the monoidal unit and the tensor preserves addition. A PET is monoidal if the subcategory of pure maps is in addition also monoidal.

Quantum Theory Reconstructed

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Let $\mathbf{C}$ be a monoidal operational PET. Then there is a functor $F : \mathbf{C} \to \mathbf{D}^{\mathrm{op}}$ with $F(\mathrm{Eff}(A)) \cong \mathrm{Eff}(F(A))$ where $\mathbf{D}$ is an appropriate category of real or complex C*-algebras.

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Recall the assumptions:

1. All maps have images.
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Yes!
(based on *Dichotomy between deterministic and probabilistic models in countably additive effectus theory*, by Cho, Westerbaan & vdW)

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Examples

$\mathbf{EJA}^{\mathrm{op}}_{psu}$, $\mathbf{vNA}^{\mathrm{op}}_{\mathrm{ncpsu}}$.

σ-effect monoids

In a σ-effect theory, the scalars $\hom(I, I)$ form a *σ-effect monoid*.

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Theorem (Westerbaan, Westerbaan & vdW, LICS'20)

A σ-effect monoid M embeds into $M_1 \oplus M_2$ where M_1 is a ω-complete Boolean algebra and $M_2 := \{f : X \to [0,1] \text{ continuous}\}$ for a basically disconnected X.

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Corollary

Scalars in a σ-effect theory are commutative.

Normalisation in σ-effect theories

Theorem

Let $\mathbf{C}$ be a σ-effect theory with $M = \hom(I, I)$.
The following are equivalent.

- States in $\mathbf{C}$ can be normalised.
- Non-zero scalars are epi.
- M has a 'division' operation.
- M has no zero divisors ($a \cdot b = 0 \implies a = 0$ or $b = 0$).
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Furthermore, if any and thus all these conditions hold then $M \cong \{0\}$, $M \cong \{0, 1\}$ or $M \cong [0, 1]$.

Dichotomy between deterministic and probabilistic models

Hence: σ-effect theories with normalisation come in three types:

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So any 'non-boring' σ-effect theory with normalisation is basically a GPT.

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- Characterize infinite-dimensional quantum theory?

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Slides available at
http://vdwetering.name/pdfs/effectus_mit_meeting.pdf

Paper at https://compositionality-journal.org/papers/compositionality-1-1/

Lecture series on Reconstructions of quantum theory
https://www.youtube.com/watch?v=-9nGxlLl614

Effectuses in Categorical Quantum Foundations
K. Cho (PhD Thesis) arXiv:1910.12198

A Characterisation of Ordered Abstract Probabilities
A. Westerbaan, B. Westerbaan, vdW arXiv:1912.10040

Dichotomy between deterministic and probabilistic models in countably additive effectus theory
K. Cho, B. Westerbaan, vdW arXiv:2003.10245

Thank you for your attention